

**Mathematical Statistics 2021/2022, Problem set 11**  
**Testing hypotheses (single sample)**

1. The diameter of rings produced by a lathe varies randomly, following a normal distribution with a standard deviation equal to 50. Based on a random sample of 25 rings, the mean diameter of rings was found to be 1025mm. Verify the hypothesis that the machine is calibrated to a diameter of 1000mm against the alternative that the diameter is greater than 1000, at a significance level of 0.02. Describe the critical region of the test used and determine the p-value for the statistic.
2. The amount of time spent by teenagers aged 15-17 on the internet is distributed normally. In a sample of 16 teens of this age group, the mean daily amount spent was approximated at 150 minutes, with a variance of 1600. Some parents believe that the average time spent daily is equal to 200 minutes. Based on the sample, is this assertion acceptable for a significance level of 0.1?
3. The weekly food expenditures of a student are random, distributed normally. On the basis of a survey of 25 students, it was asserted that the mean expenditures are equal to 250PLN, with a standard deviation of 50. For a significance level of 0.01 verify the hypothesis that the mean weekly expenditures are equal to 280 against the alternative that they are less than 280. For which significance levels would the verification yield a different result?
4. 100 random sale offers of 5-year old used cars of a specific model were analyzed. The mean price was 3.6 (thousand USD), with a sample variance of 0.64. The distribution of prices is unknown. Verify if this sample allows to assume that the mean price for this type of vehicle is equal to 3 (thousand USD), or rather more, at a 0.05 significance level?
5. A producer of cereal claims that the standard deviation of the weight of a pack of corn flakes is equal to 15g. The standard deviation from a sample of 20 random packs was found to be 19g. Assuming the weight of corn flakes in a pack is distributed normally, verify whether the result confirms the producer's claim, at a significance level of 0.05.
6. A researcher wishes to determine if the fraction of supporters of party X is equal to 20%, or more. In a sample of 1024 persons, 236 declared to be supporters. Verify the researcher's hypothesis at a significance level of 0.01. What is the p-value of the resulting statistic?