**Mathematical Statistics 2021/2022, Problem set 10**

**Neyman-Pearson Lemma**

1. We observe a nonnegative random variable and we verify the hypothesis

$$H_0 : X \text{ is distributed with density} f(x) = e^{-x}$$

versus the alternative that

$$H_1 : X \text{ is distributed with density} f(x) = xe^{-x}.$$

Build the most powerful test for significance level $\alpha = 0.05$.

2. We conduct 10 independent repetitions of an experiment, each ending successfully with the same, unknown probability $p$. Let $X$ denote the number of successes. Build the most powerful test for the verification of $H_0 : p = \frac{1}{2}$ versus $H_1 : p = \frac{3}{4}$, with the level of significance equal to $\alpha = 0.05$. What would be the most powerful test if the alternative was $\tilde{H}_1 : p = \frac{1}{4}$?

3. We have a single observation $X$ from a distribution with density $f_\theta(x) = \frac{\theta}{x^{\theta+1}}\mathbf{1}_{(1,\infty)}(x)$ with an unknown parameter $\theta > 0$. Find the critical region and the power of the test for the most powerful test to verify $H_0 : \theta = 1$ against $H_1 : \theta = 3$ for a significance level of 0.04.

4. Let $X_1, ..., X_n$ be independent random variables from a normal distribution with mean $m$ and variance $3^2$. Find the most powerful test to verify $H_0 : m = 2$ versus $H_1 : m = 4$ for a significance level of $\alpha = 0.1$. What would be the sample size needed to assure that the error of second type does not exceed 0.1? What would be the uniformly most powerful test for the verification of $H_0 : m \leqslant 2$ versus $H_1 : m > 2$? What would be the uniformly most powerful test for the verification of $H_0 : m \geqslant 2$ versus $H_1 : m < 2$?