**Mathematical Statistics 2020 /2021, Problem set 4**

**Point estimation – different techniques**

1. Let $X$ denote the number of failures before the first success in a Bernoulli scheme with probability of success equal to $\theta$, i.e. $P_\theta(X = k) = \theta(1-\theta)^k$, for $k = 0, 1, ....$ Find the method of moments estimator for $\theta$, based on the mean. What will be the precise value of the estimator, if in a sample of $n$ observations, the average number of failures is equal to 4? Find the method of moments estimator for $\theta$, based on the variance. What will be the precise value of the estimator, if in a sample of $n$ observations, the variance of the number of failures is equal to 20?

2. Let $X_1, X_2, ..., X_n$ be a random sample from a distribution with density

$$f_\theta(x) = \begin{cases} \theta x^{\theta-1} & x \in (0,1) \\ 0 & \text{otherwise} \end{cases},$$

   where $\theta > 0$ is an unknown parameter.

   Find the method of moments and method of quantiles estimators for $\theta$ (based on the mean and median, respectively) and the m.l.e. of $\theta$.

   Compare the values of the three estimators for a sample consisting of the following three observations: $\frac{1}{3}$, $\frac{1}{2}$, $\frac{5}{6}$.

3. The size of organisms from a specific population has a distribution with density $f_\beta(x) = \frac{x}{\beta^2} e^{-\frac{x}{\beta}}$ for $x > 0$ (and 0 otherwise), where $\beta > 0$ is an unknown parameter. A sample of $n$ organisms is drawn. Find the m.l.e. of $\beta$, and the precise value of the estimator if the mean size in the sample is equal to 3.

4. Let $X_1, X_2, ..., X_n$ be a random sample from a distribution with density $f_{a,b}(x) = abx^{a-1}e^{-bx^a}$ for $x > 0$ and 0 otherwise, where $b > 0$ is an unknown parameter. Find the m.l.e. of $b$.

5. Fish in a lake, again. (Unknown number of $N$ fish, $m$ caught, marked and released, $n$ caught again, among them $X$ marked. – Problem 2 from Set 3) Find the m.l.e. for the total number of fish.