Mathematical Statistics 2021 /2022, Problem set 3
The Statistical Model + Distributions of Statistics

1. There are n independent repetitions of a test. A single test ends successfully with probability θ, such that $\theta \in [\theta_1, \theta_2]$ is an unknown parameter, and θ_1 and θ_2 are known (and fall into the interval $(0, 1)$). Describe the statistical model of the overall experiment.

2. A population of N fish lives in a lake. m of them are caught, marked and released. Some time later, $n < m$ fish are caught, and X, the number of marked specimens among them, is determined. Describe the statistical model of the experiment.

3. A lottery box consists of 50 tickets, an unknown number of which are winning. 5 tickets are drawn one after the other, and the outcomes are noted. Describe the statistical model of the experiment. Would a change in the lottery scheme (when the drawn tickets are returned to the box) change the statistical model?

4. We observe the total number of insurance claims from a set of n independent identical contracts. Assume that the number of claims for a single contract is a Poisson variable with an unknown parameter λ. Describe the statistical model of the experiment.

5. A device consists of k elements of type A and l elements of type B. Assume that the duration of an element of type A follows an exponential distribution with an unknown parameter a, and the duration of an element of type B follows an exponential distribution with an unknown parameter b. The device breaks down when any of the elements stop working. We observe the life duration of the whole device. Describe the statistical model of the experiment.

6. A 6-element simple random sample is drawn from $N(\mu, 12^2)$. Determine the probability $P(13,2 < S^2 < 38,54)$, where $S^2 = \frac{1}{6}\sum_{i=1}^{6}(X_i - \bar{X})^2$.

7. A typical student spends X hours weekly reading books, where X has a normal distribution with a standard deviation of 1.5. What is the probability that the standard deviation calculated on the basis of a sample o 20 random individuals will not exceed 2?