Probability Calculus 2021/2022
Problem set 5

1. Let X be a random variable with a density function equal to
$$g(x) = 2x^{-2}1_{[2,\infty)}(x).$$
Find the CDF of variable X and the CDF of variable X^2.

2. Let F denote the CDF of a random variable X, defined by:
$$F(t) = \begin{cases} 0 & \text{if } t < -2, \\ \frac{1}{3} & \text{if } t \in [-2, 0), \\ \frac{1}{3}t + 1/2 & \text{if } t \in [0, 1), \\ \frac{5}{6} & \text{if } t \in [1, 5), \\ 1 & \text{if } t \geqslant 5. \end{cases}$$
Calculate $\mathbb{P}(X \in (3,7))$, $\mathbb{P}(X \in [-2,-1])$, $\mathbb{P}(X \in [-2,-1))$, $\mathbb{P}(X \in (-2,-1))$, $\mathbb{P}(X = 0)$, $\mathbb{P}(X = 2)$. Is the distribution of X discrete? Is the distribution of X continuous?

3. Let X be a random variable with density $g(x) = \frac{3}{8}x^2 1_{(0,2)}(x)$. Find the distributions of a) $\max\{X, 1\}$, b) X^{-2}. Are these distributions continuous? If yes, calculate the density.

4. Let X be a uniform random variable over $(0, 1)$. Find the distribution of $Y = -\ln X$.

5. Each day, an individual calls a male colleague (with probability $1/3$) or a female colleague (with probability $2/3$). The duration of a call with a male colleague is a random variable from a uniform distribution over the interval $[1, 5]$, and with a female colleague – an exponential distribution with parameter $1/5$. Let X denote the length of the telephone call on a given day. Find the distribution of random variable X and its density.

Some additional problems

Theory (you should know after the fifth lecture and before the fifth class):

1. What is the CDF of a probability distribution? What are the properties that a function must fulfill in order to be a CDF? How do we determine the type of distribution (discrete/continuous) from the CDF?

Problems (you should know how to solve after class 5)

1. Let X be a random variable with density $g(x) = \frac{1}{2}\sin x\, 1_{[0,\pi]}(x)$. Show that $\pi - X$ has the same distribution as X.

2. Let X be a random variable from a binomial distribution $B(n,p)$. Verify that $n - X$ has a binomial distribution $B(n, 1-p)$.

3. We randomly draw a point from a disk of radius R. Let X denote the distance of this point from the center of the disk. Find the distribution of X^2.

4. Let X be a random variable with density $g(x) = \frac{1}{2}x\, 1_{[0,2]}(x)$. Find the distribution of $Y = \min\{X-1, 0\}$. Does Y have a density function?