Probability Calculus 2021/2022
Problem set 4

1. Two dice were rolled. Let X denote the sum of points on the two dice. Calculate $\mathbb{P}(X \leqslant 3)$, $\mathbb{P}(X = 7)$, $\mathbb{P}(X > 10.25)$, $P(X \leqslant 1)$.

2. A die was rolled. Let X denote the number obtained, and $Y = 7 - X$. Check that X and Y have the same distribution.

3. Let k be an integer. Let X denote the number of the Bernoulli trial – in a series of Bernoulli trials with a probability of success p – where the k-th success appeared. Determine the distribution of X.

4. There are 10 balls in a box, bearing numbers from 1 to 10. We randomly draw a ball 20 times with replacement. Let X denote the smallest number obtained. Find the distribution of X and X^2.

5. Let X be a random variable with a uniform distribution over the interval $[0, 2]$. Find the density function and calculate $\mathbb{P}(X \in [1, 3])$.

6. Let X be a random variable from an exponential distribution with parameter $\lambda = 3$. Find

   (a) $\mathbb{P}(X \in [3, 4])$;

   (b) the distribution of the variable $Y = \lfloor X \rfloor$ (largest previous integer of X).

7. Let X be a random variable from an exponential distribution with parameter $\lambda > 0$, and let t and h be positive real numbers. Calculate $\mathbb{P}(X > t+h \big| X > h)$ and compare it with $\mathbb{P}(X > t)$.

8. Let X be a random variable with a density function equal to

$$g(x) = Cx^{-2}1_{[2,\infty)}(x).$$

   (a) Find C.

   (b) Calculate $\mathbb{P}(X \in [1, 12])$.

Some additional problems

Theory (you should know after the fourth lecture and before this class):

1. What is a random variable? What is the distribution of the random variable?

2. Define a discrete and a continuous distribution.

3. Define the binomial, geometric, Poisson and uniform distributions.

Problems (you should know how to solve after this class)

4. Let X be a random variable with a Poisson distribution with parameter 2. Calculate $\mathbb{P}(X = 3)$ and $\mathbb{P}(X \leqslant 2)$.

5. Let X denote the number of points obtained in a die roll. Find the distribution of $Y = X^2$.

6. Let X be a random variable uniformly distributed over $[-5, 8]$. Calculate $\mathbb{P}(X = -1)$ and $\mathbb{P}(X \leqslant 5)$.

7. Let X be a random variable with density

$$g(x) = Cx^{-3}\, 1_{[1,5]}(x) = \begin{cases} Cx^{-3} & \text{for } 1 \leqslant x \leqslant 5, \\ 0 & \text{otherwise.} \end{cases}$$

Find C and $\mathbb{P}\left(\frac{1}{X} \in [\frac{1}{2}, 3]\right)$.

8. Let X be a random variable from a geometric distribution with parameter p. let $k, l > 0$ be integer numbers. Calculate $\mathbb{P}(X > k + l | X > k)$ and compare with $\mathbb{P}(X > l)$.