

Probability Calculus 2021/2022
Problem set 3

1. Individuals I_1, I_2, \dots, I_{10} queue in random order. Verify the independence of the following pairs of events:
 - a) $\{I_1 \text{ stands directly before } I_2\}, \{I_5 \text{ stands before } I_4 \text{ (not necessarily directly)}\}$,
 - b) $\{I_1 \text{ stands before } I_2 \text{ and before } I_3\}, \{I_2 \text{ does not stand at the end}\}$.
2. We roll two dice. Let A – an event that an odd number was obtained on the first die; B – an event that an odd number was obtained on the second die; C – an event that the sum of numbers from the two dice was odd. Verify whether A, B and C are independent (as a group, and pairwise).
3. There are 5 white balls and 3 black balls in one box, and 2 white balls and 2 black balls in a second box. We repeat the following experiment 10 times: we draw a ball from each box, check if the balls are of the same color, and put them back in their respective boxes. What is the probability that we will draw balls of the same color at least twice?
4. There are six normal dice in a box, and one special die (which has sixes on two sides and ones on the remaining four sides). We randomly draw a die, and then roll it 10 times.
 - a) What is the probability that each number obtained will be a 1 or a 6?
 - b) What is the probability that we have chosen the special die, given that each number obtained was a 1 or a 6?
5. Rifleman A hits the target with probability $\frac{2}{3}$, and rifleman B with probability $\frac{3}{4}$. Six shots are made. Before each shot, a symmetric coin is flipped to determine the shooter: if the result is heads, then A shoots; if the result is tails, then B shoots.
 - a) What is the probability that the target will be hit 4 times?
 - b) The target was hit 4 times. What is the probability that only rifleman A was the shooter?
 - c) What is the most probable number of target hits?
6. An article consists of one million characters. A single character is typed wrongly with probability 0.001. An editor finds a mistake with probability 0.99. Approximate the probability that there are at most two errors in the text a) before editing and b) after editing. What is the assessment of the approximation error?
7. A dough for 1000 sweet rolls is prepared. There are 2000 raisins in the dough. Approximate the probability that in a randomly selected roll, there will be at least one raisin. Assess the approximation error.

Some additional problems

Theory (you should know after the third lecture and before this class):

1. Define independence of events. What does pairwise independence of A_1, A_2, \dots, A_{20} mean?
2. What is a Bernoulli process?
3. Formulate the Poisson theorem.

Problems (you should know how to solve after this class)

4. Two cards were drawn from a deck of 52, without replacement. Are the events {at least one club was drawn} and {no figures were drawn} independent?

5. Let A, B be events such that $\mathbb{P}(A) \in \{0, 1\}$. Prove that A and B are independent.

6. A die was rolled 10 times. What is the probability that exactly seven threes were obtained? What is the most probable number of threes in this experiment?

7. We draw 10 numbers x_1, x_2, \dots, x_{10} independently from the set $\{\frac{1}{2}, 2\}$. Calculate the probability that $x_1 x_2 \dots x_{10} = 4$.

8. 20 numbers were drawn independently from the set $[0, 3]$. Calculate the probability that exactly half of them will fall into the interval $[2, 3]$.

9. 2000 numbers were drawn independently from the interval $[0, 1000]$. Approximate the probability that at least 3 of them will fall into the interval $[20, 21]$. Assess the approximation error.