# Probability Calculus 2021/2022
## Problem set 2

1. We randomly choose a point from the interval $[0,1]$. What is the probability that it is not further from the center of the segment than $\frac{1}{4}$?

2. John and Mary agreed to meet in a cafe between 6 and 7 PM. The person who comes first waits half an hour, but then leaves. What are the chances that the pair will meet? What are the chances that John came first, given that Mary did not arrive within the first 30 minutes? What are the chances that the pair will meet, given that Mary did not arrive within the first 30 minutes?

3. John plays Texas Hold'em poker. He was given 2 cards (from a deck of 52), he examined them and did not see an ace. What is the probability that after the flop (three additional cards are shown on the table) he still won't see an ace?

4. We roll a die, and then roll it again if a six was not the outcome in the first throw. What are the chances of obtaining at least one result equal to two?

5. In a population of 1000 individuals there are (on average) 10 who are carriers of a malicious disease. There exists a test for the verification of the carrier state; the test gives a positive result for all carriers, and a (false) positive result for 5% of those who are not carriers.

   (a) What is the probability that for a randomly chosen individual the test will give the correct result?

   (b) What is the probability that for a randomly chosen individual the test will give a positive result?

   (c) What is the probability that for a randomly chosen individual, for two tests conducted (independently), both will give positive results?

   (d) What is the probability that an individual is a carrier, given that the test was positive?

   (e) What is the probability that an individual is a carrier, given that the results of two tests were both positive?

6. Among 1000 clients of an insurance company there are 20 road hogs. A road hog commits an accident with probability 0.8; a regular driver commits an accident with probability 0.2 (during the insurance period).

   (a) What are the chances that a randomly chosen driver will not commit an accident during the insurance period?

   (b) A randomly chosen driver did not commit an accident. What are the chances that he will not commit an accident during the next insurance period?

## Some additional problems

Theory (you should know before this class):

**1**. Provide the formula for conditional probability and the Law of Total Probability.
**2**. What is a partition of the sample space?
**3**. Formulate the Bayes theorem.

Problems (you should know how to solve after this class):

**4**. A die was rolled 3 times. What is the probability that no sixes appeared, if there were no twos and no fours?

**5**. $n$ individuals ($n \geqslant 3$), among which there are individuals $X$, $Y$ and $Z$, queue in random order. Calculate the probability that $X$ stands before $Y$ (not necessarily directly), if we know that $Z$ is just behind $Y$.

**6**. We have two coins: a symmetric one and one where the probability of heads is 2/3. We randomly chose a coin and toss it twice. What is the probability that results of the two tosses will be the same?

**7**. There are two white balls in one box, and one white ball and one black ball in another box. A box was chosen randomly, and then a random ball was drawn. What is the probability that the first box was chosen, given that the drawn ball was white?

**8**. A number from the interval $[0, 3]$ was drawn. Find the probability that it is smaller than 2, if we know that it is greater than 1.

**9**. We randomly draw 5 cards without replacement from a deck of 52 cards. What are the chances of obtaining exactly three aces, given that

- We did not draw a club?

- We drew at least one black ace?

- The first card was an ace?

**10**. A player received 13 cards (from a deck of 52). He examined 8 of them and noted that there were no aces. What is the probability that he will not have an ace at all?