

**Probability Calculus 2021/2022**  
**Problem set 1**

1. Events  $A$  and  $B$  are equally probable, and  $\mathbb{P}(A) = 2\mathbb{P}(A \cap B)$ . Prove that  $\mathbb{P}(B) \leq \frac{2}{3}$ .
2. From a set of numbers from 1 to 52 we randomly draw 7 numbers (without replacement). What is the probability, that
  - a) exactly 6 numbers from the range from 1 to 13 were drawn?
  - b) at least 6 numbers from the range from 1 to 13 were drawn?
  - c) no number divisible by 13 was drawn?
  - d) no number divisible by 13 was drawn or no number divisible by 4 was drawn?
  - e) the first number did not exceed 16?
3. We throw a cubic die three times. What is the probability that the same result will be obtained exactly twice?
4. The set of numbers from 1 to 52 was divided into four subsets of 13 numbers each. What is the probability that in each subset there is at least one number not exceeding 13?
5. The letters A, A, L, M, M, M, S were randomly set in a series. What is the probability that the word "MAMMALS" was obtained?
6. We draw 10 numbers, with replacement, from the set  $\{1, 2, \dots, 15\}$ . What is the probability that the maximum value obtained is 11?
7. From the set  $\{0, 1, 2, \dots, 9\}$  we draw in sequence, without replacement, four numbers:  $a, b, c, d$ . What is the probability that
  - a)  $a$  is the largest among the four?
  - b) the series  $(a, b, c, d)$  is decreasing?
8. Passengers board on an empty train with  $n$  carriages. What is the chance that there will be a passenger in each carriage, if there are
  - a)  $n$  passengers boarding?
  - b)  $n + 1$  passengers boarding?

**Some additional problems**

Theory (you should know before this class):

1. Specify the axiomatic definition of probability.
2. Define the complementary event.
3. What is classical probability?

Problems (you should know how to solve after this class):

4. Events  $A$  and  $B$  are such that  $\mathbb{P}(A) = \frac{1}{2}$ ,  $\mathbb{P}(B) = \frac{2}{3}$ ,  $\mathbb{P}(A \setminus B) = \frac{1}{3}$ . Calculate  $\mathbb{P}(B \setminus A)$ .
5. A cubic die was rolled twice. What is more probable: obtaining a sum of points equal to 7 or rolling the same number twice?

6. A coin was tossed five times. What is the probability that heads appeared at least twice?
7. The numbers 1, 2, ..., 7 were rearranged in a random sequence. What is the probability that no odd numbers are next to each other?
8. 52 cards were distributed among four players, 13 each. What is the probability that each player obtained at least one card in clubs?
9. From a deck of 52 cards we randomly draw 7 cards (without replacement). What is the probability, that
  - a) exactly 6 cards of clubs were drawn,
  - b) at least 6 cards of clubs were drawn,
  - c) no kings were drawn,
  - d) no hearts were drawn or no aces were drawn,
  - e) the first card was a heart?