Probability Calculus 2021/2021
Introductory Problem Set

1. Check that:

- $\binom{n}{k} = \binom{n}{n-k}$ and $\binom{n}{k} + \binom{n}{k+1} = \binom{n+1}{k+1}$ for $0 \leqslant k \leqslant n$;
- $\binom{n}{k} = \frac{n}{k}\binom{n-1}{k-1}$ $0 \leqslant k \leqslant n$;
- $(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$, where $a, b \geqslant 0$;
- $\sum_{k=0}^{\infty} x^k = \frac{1}{1-x}$ oraz $\sum_{k=1}^{\infty} kx^{k-1} = \frac{1}{(1-x)^2}$.

2. Calculate:

- $\sum_{k=1}^{n} k\binom{n}{k} a^k b^{n-k}$, where $a, b \geqslant 0$;
- $\sum_{k=0}^{\infty} \frac{\lambda^k x^k}{k!}$ and $\sum_{k=0}^{\infty} k\frac{\lambda^k x^k}{k!}$, for $0 < x < 1$.

3. Find

- $\int_0^{\infty} \exp(-ax)dx$, where $a > 0$;
- $\int_0^{\infty} xe^{-ax}dx$, where $a > 0$;
- $\int_0^1 x^p dx$ where $p > -1$;
- $\int_1^{\infty} x^{-p}dx$, where $p > 1$;
- $\int_0^{\infty} \frac{1}{1+x^2}dx$.
- $\int_0^{\pi} \sin x dx$.

4. Let A, B, C be events. Using the notation with operations on sets, how would you write "exactly two among events A, B and C occurred"?

5. Explain what we mean by

- variations with repetitions
- variations without repetitions
- permutations
- combinations