Probability Calculus

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lecture I, 7.10.2021

INTRODUCTION TO PROBABILITY CALCULUS

Some technicalities

- Contact: ajanicka@wne.uw.edu.pl
- Office hours: Tue, 12:00 AM at the Faculty, or email me for a ZOOM link
- web page (materials):
  www.wne.uw.edu.pl/ajanicka/pc
- Readings: **LECTURE NOTES** &
  - **Charles M. Grinstead and J. Laurie Snell, Introduction to Probability, available online**
  - **Sheldon M. Ross, Introduction to Probability Models, available in the FoES library and online**
  - **Wackerly, D., Mendenhall, W., & Scheaffer, R. Mathematical statistics with applications, available in the FoES library**

Assessment

1. Presence during lectures – recommended; **presence during Wednesday classes – mandatory**
2. Class assessment: two tests (2 x 30p)+ team activity during classes (30p) + individual activity in moodle (30p)
3. Teamwork!
4. Homework
5. Final grade: class points (40pts) + final exam (50pts) + homework (10pts) for those who pass classes

What to expect

- Lecture notes (web page)
- Problems to solve: teamwork & during classes (web page) + moodle links
- Homework (web page) + moodle links
- Activities (moodle)

Thematic scope of course

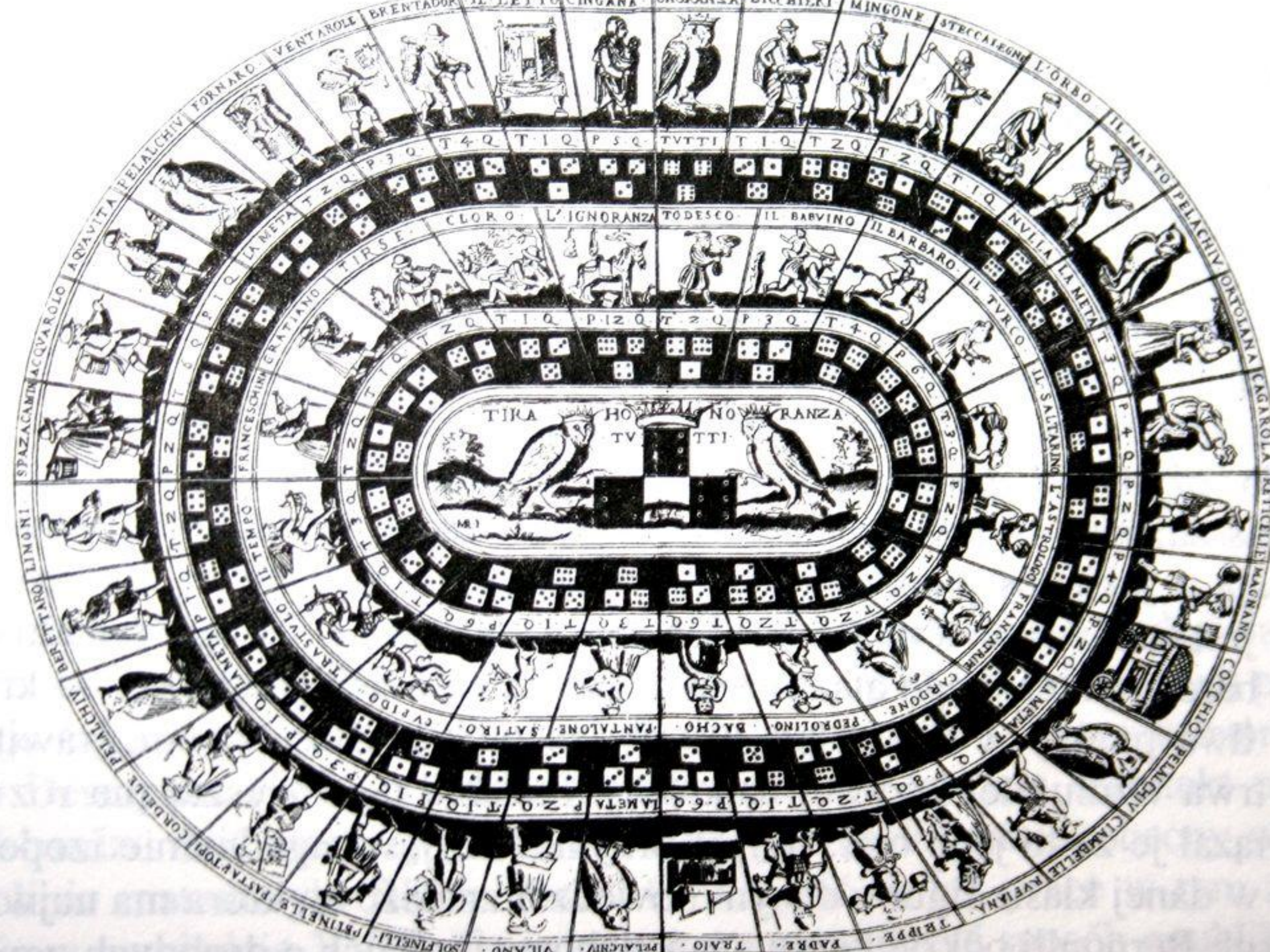
- Some basics and „classics”
- „Contemporary” probability
- Reality description – random variables. Crucial in statistics and econometrics
- Limit theorems – crucial as above, very important in practice (e.g. insurance)

Plan for today

1. Historical perspective
2. Basic definitions and notations, examples
3. *σ-algebras*
4. Probability intuitively and **Kolmogorov axioms,**
5. Examples
6. Basic properties of probability

1. Historical perspective

- Motivation:
  - gambling
  - statistics of births and deaths
  - insurance of transports
- „Paradoxes”
- First mathematical publications without errors: Bernoulli, 1752
- „Contemporary probability”: Kolmogorov axioms, 1933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FORNARO
VENTAROLE
CLORO
L'IGNORANZA
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IL BABVINO
IL BARBARO
IL TVRCO
IL SALTARINO
L'ASTROLOGO
FRANCATRIPE
CARDONE
PEDROLINO
BACHO
PANTALONE
SATIRO
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1. Historical perspective

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Gerolamo Cardano, De Ludo Aleae (Book on Games of Chance), 1564

"If it is necessary for someone that he should throw at least twice, then you know that the throws favorable for it are 91 in number, and the remainder is 125; so we multiply each of these numbers by itself and get to 8281 and 15625, and the odds are about 2 to 1."

*"**This reasoning seems to be false**... for example, the chance of getting one of any three chosen faces in one cast of one dice is equal to the chance of getting one of the other three, but according to this reasoning there would be an even chance of getting a chosen face each time in two casts, and thus in three, and four, **which is most absurd**."*

Faculty of Economic Sciences

1. Historical perspective

- Motivation:
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2. Basic definitions and notations

Elementary event: ω

Sample space : Ω

Event: *A, B, etc.*

Special events, operations:

Ω, , A', $A \cup B$, $A \cap B$, $A \setminus B$, $A \subseteq B$

2. Examples

1. Coin toss
2. Dice rolling
3. Rolling of a pair of dice – sum of points
4. Eurojackpot: draw of 5 numbers out of 50 – with and without order
5. Coin toss until first „heads”
6. A geometric example

3. σ-algebra

Defines the sets that we can measure (calculate probability). In most simple cases: **we don't need to worry about it.**

Definition of a σ-algebra $\mathscr{F}$ of subsets of Ω

A family $\mathcal{F}$ of subsets of Ω is called a σ-algebra, *if*

$$
\begin{aligned}
&(i) \quad \emptyset \in \mathcal{F}, \\
&(ii) \quad A \in \mathcal{F} \Rightarrow A' \in \mathcal{F}, \\
&(iii) \quad A_1, A_2, \ldots \in \mathcal{F} \Rightarrow \bigcup_{n=1}^{\infty} A_n \in \mathcal{F}.
\end{aligned}
$$

4. Probability intuitively – frequencies

$$\rho_n(A) = \frac{\text{number of occurrences of } A}{n}$$

- Calculating frequencies
- Properties of frequencies
- Limit =?

5. Probability formally - Kolmogorov Axioms

- For a given $(\Omega, \mathcal{F})$ we define probability as a function satisfying 3 conditions

$$
\begin{aligned}
&(i) && 0 \leq \mathbb{P}(A) \leq 1, \\
&(ii) && \mathbb{P}(\Omega) = 1, \\
&(iii) && \text{if } A_1, A_2, \ldots \in \mathcal{F} \text{ are pairwise disjoint} \\
& && \mathbb{P}\left(\bigcup_{n=1}^{\infty} A_n\right) = \sum_{n=1}^{\infty} \mathbb{P}(A_n).
\end{aligned}
$$

- Probability space $(\Omega, \mathcal{F}, \mathbb{P})$

6. Examples

1. Symmetric coin toss, asymmetric coin toss
2. Dice rolling
3. **Classic scheme (simple probability)**
4. Drawing numbers (Eurojackpot)
5. Geometric probability

7. Basic properties of probability

- Theorem 1 (arithmetics)

Theorem 1. *Let* A, B, A_1, A_2, $\ldots \in \mathcal{F}$. *Then*

(i) $\mathbb{P}(\emptyset) = 0$,

(ii) *If* $A_1, A_2, \ldots, A_n$ *are pairwise disjoint, then* $\mathbb{P}\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} \mathbb{P}(A_i)$.

(iii) $\mathbb{P}(A') = 1 - \mathbb{P}(A)$.

(iv) *If* $A \subseteq B$, *then* $\mathbb{P}(B \setminus A) = \mathbb{P}(B) - \mathbb{P}(A)$.

(v) *If* $A \subseteq B$, *then* $\mathbb{P}(A) \leqslant \mathbb{P}(B)$.

(vi) $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B)$.

(vii) $\mathbb{P}\left(\bigcup_{i=1}^{\infty} A_i\right) \leqslant \sum_{i=1}^{\infty} \mathbb{P}(A_i)$.

7. Basic properties of probability – cont.

- Theorem 2 (inclusion-exclusion principle)

$$\textit{If } A_1, A_2, \ldots, A_n \in \mathcal{F}, \textit{ then}$$

$$\begin{aligned}\mathbb{P}(A_1 \cup A_2 \cup \ldots \cup A_n) = & \sum_{i=1}^{n} \mathbb{P}(A_i) - \sum_{i<j} \mathbb{P}(A_i \cap A_j) + \ldots \\ & + (-1)^{n+1} \mathbb{P}(A_1 \cap A_2 \cap \ldots \cap A_n)\end{aligned}$$

Comments?

https://jamboard.google.com/d/1W9kvVtwIFEv3FjEplHJivDzXPQcho5QVQgLXlCgmfcM/edit?usp=sharing