Mathematical Statistics 2020/2021, Problem set 15
Hypothesis testing – revision

1. A sample of 16 car users were interrogated in order to determine the number of kilometers traveled daily by car. In the studied sample, the average amounted to 24, with a standard deviation (based on the unbiased estimator of the variance) equal to 9. We assume that distances traveled follow a normal distribution.
   - We verify the null hypothesis that car users travel on average 25 km daily against the alternative that they travel less, for a significance level of 0.1. The value of the appropriate test statistic is equal to ...., the critical region of the appropriate test is equal to ...., so we REJECT /have NO GROUNDS TO REJECT (underline the appropriate) the null hypothesis.
   - We verify the null hypothesis that the variance of daily travels amounts to 100, against the alternative that it is smaller, for a significance level of 0.1. The value of the appropriate test statistic is equal to ...., the critical region of the appropriate test is equal to ...., so we REJECT /have NO GROUNDS TO REJECT (underline the appropriate) the null hypothesis.
2. A streaming service provider granted access to its services in a new country, where the fraction of individuals in the population who speak English is lower than average and wishes to determine whether this fact has impact on the number of movies that the users watch. In the new country, in a sample of 36 new subscribers, the average monthly number of movies watched was 6.4. In general, the streaming service provider observed on average 8 movies watched per user per month, with a standard deviation equal to 3. We make a simplifying assumption that the number of movies watched follows a normal distribution.
   - The value of the appropriate test statistic is equal to ...., the p-value of this result is equal to ...., so for a 10% significance level we REJECT /DO NOT HAVE GROUNDS TO REJECT (underline the appropriate) the null hypothesis that the average for the new users is the same as in general.
   - Additionally, it was calculated that the value of the standard deviation in the new country sample amounted to 2.7. We verify whether the variance of the number of movies watched in the new country is the same as in the general population for a significance level of 10%. The value of the appropriate test statistic amounts to .... The critical VALUE of the appropriate test amounts to .... and comes from a distribution with .... degrees of freedom. Based on these results, we REJECT /DO NOT HAVE GROUNDS TO REJECT (underline the appropriate) the null hypothesis.
3. In some countries, a streaming service provider offers an additional option of movie subtitles in the native language of the subscribers, but in some countries this feature is not available. A survey conducted with random 1000 users in each of the two groups of countries showed that in countries where native language subtitles are available, the average amount of time spent on receiving streams amounts to 300 minutes weekly, with a standard deviation equal to 50 minutes, while in countries where the native language subtitles are not available, the average amount of time spent amounts to 200 minutes, with a standard deviation equal to 40 minutes.
   - We conduct a test for the equality of mean stream duration for the two groups, against the alternative that in the group without subtitles the average is lower. The value of the appropriate test statistic is equal to ...., the critical REGION of the appropriate test for a 1% significance level is equal to ...., so for this significance level we REJECT /DO NOT HAVE GROUNDS TO REJECT (underline the appropriate) the null hypothesis.
   - Let us assume that the same values of averages and standard deviations were obtained for sample sizes of 10000 users in each of the two groups, respectively. In such a case, the value of the test statistic calculated for the larger sample would be HIGHER /LOWER /THE SAME (underline the appropriate) as in the previous point, while the p-value of the test statistic calculated for the larger sample would be HIGHER /LOWER /THE SAME (underline the appropriate).
4. A streaming service provider wishes to determine whether the length of a free trail period translates to a tendency in paid subscriptions. The data from a randomized sample is summarized in the table below:

Length of free trial period offered, in months	1	2	3	4
Number of users who continue with paid subscription	60	70	50	50
Number of users who do not continue with paid subscription	40	30	50	50
Number of users who were offered such trial length	100	100	100	100

- We conduct a chi-square test of independence to determine whether the length of the free trial period determines the willingness to continue with a paid subscription. The value of the appropriate chi-square test statistic amounts to ...., the critical REGION for a 5% significance test is equal to ...., so for this significance level we REJECT /DO NOT HAVE GROUNDS TO REJECT (underline the appropriate) the null hypothesis of the lack of influence of the length of the free trial period on paid subscriptions.
- We conduct a simple test to compare whether the fractions of users who continue with a paid subscription are the same for the one month and two months free trial periods. The value of the appropriate test statistic amounts to ...., the critical VALUE for a 5% significance test is equal to ...., so for this significance level we REJECT /DO NOT HAVE GROUNDS TO REJECT (underline the appropriate) the null hypothesis of the equality of fractions.

5. A HR specialist is especially interested in two traits of workers: whether they are diligent (or lazy) and whether they are creative (or not). We assume that the two traits are independent, and that the probabilities that a worker is diligent and lazy are each equal to $\frac{1}{2}$. We assume that the probability that a worker is creative is equal to p, where $p \in (0, 1)$ is unknown. A group of workers were asked to take a test; the results are summarized in the table below:

Type of worker	Diligent and creative	Diligent and not creative	Lazy and creative	Lazy and not creative
Number of workers	$N_1 = 10$	$N_2 = 30$	$N_3 = 20$	$N_4 = 40$
Assumed probabilities	$\frac{1}{2}p$	$\frac{1}{2}(1-p)$	$\frac{1}{2}p$	$\frac{1}{2}(1-p)$

- Find the formula for the MLE of p (depending on N_1, N_2, N_3, N_4):

  $\hat{p} =$ .... and the value of the estimator for the above sample: ....
- Using the chi-square test (and the estimate from the previous point), verify whether the assumed probability scheme is valid for a significance level 0.05. The value of the appropriate test statistic is equal to ...., under the null hypothesis the statistic has a distribution with .... degrees of freedom and a critical value equal to ...., so we REJECT /HAVE NO GROUNDS TO REJECT (underline the appropriate) the null hypothesis.

6. We analyze the wages of workers, depending on their traits. In a random sample of workers, the average wage for a group of 200 workers identified as lazy amounted to \$ 2400, with a standard deviation equal to \$ 500. Meanwhile, for a group of 200 workers identified as diligent, the average amounted to \$ 2600, with a standard deviation also equal to \$ 500. Standard deviations were calculated on the base of the unbiased estimator of the variance.

- We want to verify whether the average wages in the two groups are the same. The value of the appropriate test statistic is equal to ...., the critical region of the test for a significance level $\alpha = 0.01$ has the form ...., so we REJECT /HAVE NO GROUNDS TO REJECT (underline the appropriate) the null hypothesis for this significance level.
- If the same averages and variances were found for a sample consisting of 8 lazy workers and 8 diligent workers, the result of a testing procedure analogous to the previous point WOULD BE THE SAME /WOULD BE DIFFERENT /WE CAN'T SAY – IT WOULD DEPEND ON THE ASSUMED DISTRIBUTION OF WAGES (underline the appropriate).

7. A social networking service company wishes to determine whether the type (a static picture or a film) and the frequency (single per visit or changing with each reload) of advertising content shown to users affects the duration of time spent using the service. The collected data for a random sample of users are summarized in the table below.

Advertising settings	Static ad, once	Static ad, changes	Film ad, once	Film ad, changes
Average time spent (hours)	1.6	1	1.2	1
Variance of time spent (biased estimator)	0.16	0.25	0.16	0.25
Number of users	20	20	20	20

We assume that the time spent using the service follows a normal distribution. We conduct an analysis of variance test for the four ways of advertising to verify the null hypothesis that they way advertising is effected does not impact the average time spent using the service.

- The sum of squares between groups (SSB) is equal to ...., and the sum of squares within groups (SSW) is equal to .... The value of the appropriate test statistic is equal to ....
- The critical region for a 5% significance level is equal to ...., so for this significance level we REJECT /DO NOT HAVE GROUNDS TO REJECT the null hypothesis (underline the appropriate). Based on this result we can say that THE TYPE /THE FREQUENCY /NEITHER THE TYPE NOR THE FREQUENCY /THE TYPE AND THE FREQUENCY COMBINED have impact on the duration of service usage (underline the appropriate).

8. We want to verify whether the duration of unemployment (in years) in a certain town can be described by a $\Gamma(1, 2)$ distribution, against the alternative that the distribution is $\Gamma(3, 2)$, on the base of a single observation. We construct the most powerful test for these two hypotheses, for a significance level $\alpha = 0.2$.

   - The critical region of the most powerful test in this case is equal to ....
   - If the observed duration of unemployment was equal to 1 month, the decision would be to REJECT /NO GROUNDS TO REJECT (underline the appropriate) the null hypothesis, and if the observed value was equal to 2 months, the decision would be to REJECT /NO GROUNDS TO REJECT (underline the appropriate) the null hypothesis.

Hint. The density of a $\Gamma(k, \lambda)$ distribution is equal to $f(x) = \frac{\lambda^k}{(k-1)!}x^{k-1}e^{-\lambda x}$ for $x > 0$, for integer values of k and $\lambda > 0$.