

Mathematical Statistics 2020/2021, Problem set 13

Testing hypotheses (multiple-sample)

- The distribution of the time devoted to individual study by undergraduate students may be described by a normal distribution: $N(m, 5^2)$; for graduate students, the distribution is $N(n, 6^2)$. A sample of 10 undergraduate and 18 graduate students were interrogated. The average study times for the two groups were equal to 20 and 15 hours, respectively. Can we accept the hypothesis that the study time of the two groups is equal, at a significance level of $\alpha = 0.1$? What is the p -value of the test statistic obtained?

- A wholesaler examined the weight of apples provided by two suppliers, and obtained the following results:

| | | |
|-------------|--------------------------------|------------------------------------|
| supplier I | $\sum_{i=1}^{10} X_{1i} = 645$ | $\sum_{i=1}^{10} X_{1i}^2 = 41715$ |
| supplier II | $\sum_{i=1}^{10} X_{2i} = 680$ | $s^2 = 10$ |

Assuming normal distributions with equal variances, verify the hypothesis that the mean weights of apples from the two suppliers are equal, at a significance level of $\alpha = 0.05$.

- A manager wishes to determine whether there are differences in the absences of workers in two factories. In a group of 180 random workers of the first factory, the mean absence in work during the analyzed year was equal to 16 days, with a standard deviation of 6. In a group of 200 random workers in the second factory, the mean yearly absence was equal to 12 days, with a standard deviation of 4. Using a significance level of 0.05, verify the hypothesis of equal average non-attendance in both factories.

- An analyst verifies a null hypothesis that the variances of prices of financial instruments A and B are equal, against the alternative that the variance of instrument A is greater. A random sample of 26 daily quotations of instrument A resulted in a standard deviation of 2.55, and a random sample of 21 prices of instrument B gave a result of 1.86. Assuming the prices are distributed normally and the measurements were independent, verify the hypothesis at a significance level $\alpha = 0.01$.

- Out of 200 characters typed by typist A, 20 were incorrect; typist B made 18 mistakes in a series of 150 characters. At a significance level of $\alpha = 0.1$ verify whether the two typists have an identical ratio of mistakes.

- The average cost of coffee in three cities was calculated, on the basis of samples sized 10 in each city. The averages were, respectively, 10.5, 9.1, 8.3, and $\sum_{i=1}^3 \sum_{j=1}^{10} (x_{ij} - \bar{x}_i)^2 = 2.7$. Applying a test of analysis of variance, verify whether the data confirms a relationship between the city and price levels. What are the necessary assumptions allowing the use of ANOVA?

- A producer analyzed the prices of a box of his chocolates in retail stores in three cities. The results are summarized in the following table:

| City | Sample size | Average price | Variance = $\frac{1}{n} \sum_{i=1}^n (X_i - \bar{X})^2$ |
|------|-------------|---------------|---|
| A | 10 | 10 | 3 |
| B | 15 | 12 | 2 |
| C | 5 | 8 | 4 |

Under the assumption that prices are independent and distributed normally with equal variances, verify the null hypothesis that the average prices in the three cities are equal, for a significance level of $\alpha = 0.01$.