

Mathematical Statistics 2020/2021, Homework 5 (Two problems)

Name and Surname ..... Student's number .....

*In the problems below, please use the following: as  $k$  – the sum of digits in your student's number; as  $m$  – the sum of the two largest digits in your student's number; and as  $n$  – the smallest digit in your student's number plus 1. For example, if an index number is 609999:  $k = 42$ ,  $m = 18$ ,  $n = 1$ .*

*Please write down the solutions (transformations, substitutions etc.), and additionally provide the final answer in the space specified (the answer should be a number in decimal notation, rounded to four digits).*

9. The amount of time it takes a student to solve a homework problem in mathematical statistics (in minutes) follows a normal distribution with unknown mean  $\mu$  and a variance equal to  $m^2$ . We want to verify the null hypothesis that  $\mu = k$  against the alternative that  $\mu = 2k$  on the base of a single observation  $X$  using the following procedure: if  $X > c$ , we reject the null. Find  $c$  such that the test has a significance level of  $n\%$ . Calculate the probability of committing an error of the second type for this test.

ANSWER:

|                |                             |
|----------------|-----------------------------|
| value of $c$ : | prob. of error of 2nd kind: |
|----------------|-----------------------------|

Solution:

**10.** The amount of time it takes a student to solve a homework problem in mathematical statistics (in minutes) follows a normal distribution with unknown mean  $\mu$  and a variance equal to  $m^2$ . Find the most powerful test to verify the null hypothesis that  $\mu = 2k$  against the alternative that  $\mu = \frac{3}{2}k$  on the base of  $k$  independent observations, at a significance level of  $n\%$ . Calculate the power of this test (for the alternative hypothesis). What is the decision, based on a sample of observations equal to  $k + 5, k + 6, \dots, 2k + 4$ ?

ANSWER:

|                                 |                       |                           |
|---------------------------------|-----------------------|---------------------------|
| critical region<br>of the test: | Power<br>of the test: | Reject null?<br>(YES/NO): |
|---------------------------------|-----------------------|---------------------------|

Solution: