

Mathematical Statistics 2020/2021, Homework 3 (two problems)

Name and Surname Student's number

In the problems below, please use the following: as k – the sum of digits in your student's number; as m – the sum of the two largest digits in your student's number; and as n – the smallest digit in your student's number plus 1. For example, if an index number is 609999: $k = 42$, $m = 18$, $n = 1$.

Please write down the solutions (transformations, substitutions etc.), and additionally provide the final answer in the space specified (the answer should be a number in decimal notation, rounded to four digits).

5. Let X_1, X_2, \dots, X_k be a random sample from a distribution with density

$$f_{\theta}(x) = m\theta^m x^{-(m+1)} \mathbf{1}_{[\theta, \infty)}(x),$$

where $\theta > 0$ is an unknown parameter.

Let $\hat{\theta}_{ML}$ be the Maximum Likelihood Estimator of θ .

a) For $\hat{\theta}_{ML}$, calculate the bias of the estimator, assuming that the true value of parameter θ is equal to $n \cdot m \cdot k$;

b) For $\hat{\theta}_{ML}$, calculate the variance of the estimator, assuming that the true value of parameter θ is equal to $n \cdot m \cdot k$.

ANSWER:

a) bias of $\hat{\theta}_{ML}$:	b) variance of $\hat{\theta}_{ML}$:
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Solution:

6. Let X_1, X_2, \dots, X_k be a random sample from a uniform distribution over the interval $[0, m\theta]$, where $\theta > 0$ is an unknown parameter, and let

$$T = c(X_1 + X_2 + \dots + X_k)$$

be an estimator of θ .

- a) Find c such that the estimator T is unbiased.
- b) Calculate the MSE of the unbiased estimator T , assuming that the true value of parameter θ is equal to $n \cdot m$.

a) value of c :

b) MSE of unbiased T :

Solution: