

**Mathematical Statistics 2020 /2021, Problem set 4**  
**Point estimation – different techniques**

1. Let  $X$  denote the number of failures before the first success in a Bernoulli scheme with probability of success equal to  $\theta$ , i.e.  $P_\theta(X = k) = \theta(1 - \theta)^k$ , for  $k = 0, 1, \dots$ . Find the method of moments estimator for  $\theta$ , based on the mean. What will be the precise value of the estimator, if in a sample of  $n$  observations, the average number of failures is equal to 4? Find the method of moments estimator for  $\theta$ , based on the variance. What will be the precise value of the estimator, if in a sample of  $n$  observations, the variance of the number of failures is equal to 20?
2. Let  $X_1, X_2, \dots, X_n$  be a random sample from a distribution with density

$$f_\theta(x) = \begin{cases} \theta x^{\theta-1} & x \in (0, 1) \\ 0 & \text{otherwise} \end{cases},$$

where  $\theta > 0$  is an unknown parameter.

Find the method of moments and method of quantiles estimators for  $\theta$  (based on the mean and median, respectively) and the m.l.e. of  $\theta$ .

Compare the values of the three estimators for a sample consisting of the following three observations:

$$\frac{1}{3}, \frac{1}{2}, \frac{5}{6}.$$

3. The size of organisms from a specific population has a distribution with density  $f_\beta(x) = \frac{x}{\beta^2} e^{-\frac{x}{\beta}}$  for  $x > 0$  (and 0 otherwise). A sample of  $n$  organisms is drawn. Find the m.l.e. of  $\beta$ , and the precise value of the estimator if the mean size in the sample is equal to 3.
4. Let  $X_1, X_2, \dots, X_n$  be a random sample from a distribution with density  $f_{a,b}(x) = abx^{a-1}e^{-bx^a}$  for  $x > 0$  and 0 otherwise. Find the m.l.e. of  $b$ .
5. Fish in a lake, again. (Unknown number of  $N$  fish,  $m$  caught, marked and released,  $n$  caught again, among them  $X$  marked.) Find the m.l.e. for the total number of fish.