

Mathematical Statistics 2020/2021, Homework 2 (two problems)

Name and Surname Student's number

In the problems below, please use the following: as k – the sum of digits in your student's number; as m – the largest digit in your student's number; and as n – the smallest digit in your student's number plus 1. Please write down the solutions (transformations, substitutions etc.), and additionally provide the final answer in the space specified (the answer should be a number in decimal notation, rounded to four digits).

3. Let X_1, X_2, \dots, X_{2n} be a sample from a distribution with density

$$f_{\theta}(x) = \begin{cases} \frac{m\theta}{k^{m\theta}} x^{m\theta-1} & x \in [0, k] \\ 0 & \text{otherwise} \end{cases},$$

where $\theta > 0$ is an unknown parameter.

Calculate the values of the Method of Moments Estimator of θ (using the mean) and the Maximum Likelihood Estimator of θ for a sample consisting of the following observations: $X_1 = k - 1, X_2 = k - 2, \dots, X_{2n} = k - 2n$.

ANSWER:

$\hat{\theta}_{MM} =$	$\hat{\theta}_{ML} =$
-----------------------	-----------------------

Solution:

4. Let X_1, X_2, \dots, X_{25} be a sample from a distribution with CDF

$$F_{\alpha,\beta}(x) = \begin{cases} 1 - e^{-\frac{\alpha}{m}x^{\beta+n}} & x \geq 0 \\ 0 & \text{otherwise} \end{cases},$$

where $\alpha > 0$ and $\beta > -n$ are unknown parameters.

Calculate the values of the Method of Quantiles Estimators of α and β (using the first and third quartiles) for a sample consisting of the following observations: $X_1 = k, X_2 = \frac{k}{2}, \dots, X_{25} = \frac{k}{25}$.

ANSWER: $\hat{\alpha}_{MQ} =$

$\hat{\beta}_{MQ} =$

Solution: