Probability Calculus Midterm Test. December 3rd, 2019. Version A

In order to obtain the maximum number of points, you need to solve 5 out of the 7 problems below. Each problem must be solved on a separate piece of paper. Please sign each paper with your name and student's number and note the version (A, B, C, D). Each problem will be graded on a scale from 0 to 10 points. Duration: 120 minutes.

1. 9 donuts and 3 muffins were laid out on 4 plates in such a way that there are three cakes on each plate.

a) Calculate the probability that there are three muffins on one of the plates.

b) Assume that it's not the case that there are three muffins on one plate. From a randomly chosen plate we move one randomly chosen cake to a different plate. Calculate the probability that after this move, the three muffins will be on the same plate.

c) Calculate the expected value of the number of muffins on the first plate.

2. There are two types of lottery tickets available in a kiosk: one, where a ticket costs \$1 and the probability of winning amounts to $\frac{1}{100}$ and one, where the ticket costs \$2 and the probability of winning amounts to $\frac{1}{50}$. Mr Smith buys one randomly chosen lottery ticket every day (we assume that tickets from the two lotteries are equally numerous on all days); the choices on different days are independent.

a) Calculate the probability that during three days Mr Smith spent no more than \$4, if we know that during this period he did not win.

b) Knowing that during seven days Mr Smith spent \$12 on lottery tickets, calculate the expected value of the number of wins during this period.

c) Using the Poisson theorem, approximate the probability that during 100 days Mr Smith will win at least twice.

3. Each day, Mr Jones drinks a certain amount of coffee: zero, one, two or three cups. The chance that he will not drink any at all is the same as the chance that he will drink three cups and is twice as high as the chance that he will drink exactly one cup. On average, Mr Jones drinks 1.5 cups of coffee per day.

a) Let X denote the number of cups of coffee that Mr Jones drinks on a given day. Find the distribution and the variance of the variable X.

b) A single coffee at a café costs \$3. Mr Jones pays jointly for all the coffees he drinks on a given day, but he only carries ten dollar bills in his wallet. With probability $\frac{1}{2}$ the waiter has only two-dollar coins. In case the waiter can't give back the full change, Mr Jones leaves the remaining dollar as a tip. Find the expected value of the total amount of tips left by Mr Jones over the period of 30 days.

4. Let X be a random variable with a cumulative distribution function:

$$F(t) = \begin{cases} 0 & \text{if } t < 1, \\ \frac{t}{3} & \text{if } t \in [1, 2), \\ \frac{t-2}{6} + \frac{2}{3} & \text{if } t \in [2, 4), \\ 1 & \text{if } t \geq 4, \end{cases}$$

and $Y = X^3$. Find the means of variables X and Y, the median of X and the first quartile of variable Y.

5. Let X be a random variable from an exponential distribution with parameter 2. Calculate the variance of variable $Z = 4X + 5$. Is variable $Y = \frac{1}{X}$ continuous? If yes, find the density.

6. X is a random variable with a mean equal to 1 and a variance equal to $\frac{1}{4}$. Let Y be a random variable from a distribution with density $f(x) = \left(a(x - \frac{m}{2})^2 + c\right) \mathbf{1}_{[0,m]}(x)$. Find values of parameters a, c and m such that the variables X and Y have the same means and variances.

7. A student has 100 hours she may devote, in any proportion, to prepare for exams from two courses. If t_1 (respectively, t_2) is the time (in hours) devoted to the preparations for the first (respectively, second) course, the probability of passing this course in one attempt amounts to $\frac{t_1}{100}$ (respectively, $\frac{t_2}{225}$). The student may retake the exams multiple times, until they are passed (all trials are independent), but she only prepares once. What should the number of hours devoted to the preparation for the first course exam be equal to, in order to minimize the average number of trials needed to pass exams from both courses?

Probability Calculus Midterm Test. December 3rd, 2019. Version B

In order to obtain the maximum number of points, you need to solve 5 out of the 7 problems below. Each problem must be solved on a separate piece of paper. Please sign each paper with your name and student's number and note the version (A, B, C, D). Each problem will be graded on a scale from 0 to 10 points. Duration: 120 minutes.

1. 12 donuts and 3 muffins were laid out on 5 plates in such a way that there are three cakes on each plate.

a) Calculate the probability that there is a plate with at least two muffins on it.

b) Assume that each muffin is on a different plate. From a randomly chosen plate we move one randomly chosen cake to a different plate. Calculate the probability that after this move, each muffin will still be on a different plate.

c) Calculate the expected value of the number of muffins on the second plate.

2. There are two types of lottery tickets available in a kiosk: one, where a ticket costs \$2 and the probability of winning amounts to $\frac{1}{120}$ and one, where the ticket costs \$4 and the probability of winning amounts to $\frac{1}{60}$. Mr Smith buys one randomly chosen lottery ticket every day (we assume that tickets from the two lotteries are equally numerous on all days); the choices on different days are independent.

a) Calculate the probability that during three days Mr Smith spent no more than \$8, if we know that during this period he did not win.

b) Knowing that during seven days Mr Smith spent \$26 on lottery tickets, calculate the expected value of the number of wins during this period.

c) Using the Poisson theorem, approximate the probability that during 120 days Mr Smith will win at least once.

3. Each day, Mr Jones drinks a certain amount of coffee: zero, one, two or three cups. The chance that he will not drink any at all is the same as the chance that he will drink three cups and is two times lower than the chance that he will drink exactly one cup. On average, Mr Jones drinks 1.5 cups of coffee per day.

a) Let X denote the number of cups of coffee that Mr Jones drinks on a given day. Find the distribution and the variance of the variable X.

b) A single coffee at a café costs \$3. Mr Jones pays jointly for all the coffees he drinks on a given day, but he only carries ten dollar bills in his wallet. With probability $\frac{1}{3}$ the waiter has only two-dollar coins. In case the waiter can't give back the full change, Mr Jones leaves the remaining dollar as a tip. Find the expected value of the total amount of tips left by Mr Jones over the period of 15 days.

4. Let X be a random variable with a cumulative distribution function:

$$F(t) = \begin{cases} 0 & \text{if } t < 1, \\ \frac{t-1}{3} & \text{if } t \in [1, 2), \\ \frac{t-2}{6} + \frac{1}{3} & \text{if } t \in [2, 4), \\ 1 & \text{if } t \geq 4, \end{cases}$$

and $Y = X^2$. Find the means of variables X and Y, the median of X and the third quartile of variable Y.

5. Let X be a random variable from an exponential distribution with parameter 3. Calculate the variance of variable $Z = 3X - 4$. Is variable $Y = \frac{2}{X}$ continuous? If yes, find the density.

6. X is a random variable with a mean equal to 1 and a variance equal to $\frac{1}{5}$. Let Y be a random variable from a distribution with density $f(x) = \left(a(x - \frac{m}{2})^2 + c\right) \mathbf{1}_{[0,m]}(x)$. Find values of parameters a, c and m such that the variables X and Y have the same means and variances.

7. A student has 96 hours she may devote, in any proportion, to prepare for exams from two courses. If t_1 (respectively, t_2) is the time (in hours) devoted to the preparations for the first (respectively, second) course, the probability of passing this course in one attempt amounts to $\frac{t_1}{100}$ (respectively, $\frac{t_2}{196}$). The student may retake the exams multiple times, until they are passed (all trials are independent), but she only prepares once. What should the number of hours devoted to the preparation for the first course exam be equal to, in order to minimize the average number of trials needed to pass exams from both courses?

Probability Calculus Midterm Test. December 3rd, 2019. Version C

In order to obtain the maximum number of points, you need to solve 5 out of the 7 problems below. Each problem must be solved on a separate piece of paper. Please sign each paper with your name and student's number and note the version (A, B, C, D). Each problem will be graded on a scale from 0 to 10 points. Duration: 120 minutes.

1. 15 donuts and 3 muffins were laid out on 6 plates in such a way that there are three cakes on each plate.

a) Calculate the probability that there is a plate with at least two muffins on it.

b) Assume that each muffin is on a different plate. From a randomly chosen plate we move one randomly chosen cake to a different plate. Calculate the probability that after this move, each muffin will still be on a different plate.

c) Calculate the expected value of the number of donuts on the first plate.

2. There are two types of lottery tickets available in a kiosk: one, where a ticket costs \$2 and the probability of winning amounts to $\frac{1}{120}$ and one, where the ticket costs \$3 and the probability of winning amounts to $\frac{1}{80}$. Mr Smith buys one randomly chosen lottery ticket every day (we assume that tickets from the two lotteries are equally numerous on all days); the choices on different days are independent.

a) Calculate the probability that during three days Mr Smith spent at least \$8, if we know that during this period he won three times.

b) Knowing that during seven days Mr Smith spent \$19 on lottery tickets, calculate the expected value of the number of wins during this period.

c) Using the Poisson theorem, approximate the probability that during 96 days Mr Smith will win at least once.

3. Each day, Mr Jones drinks a certain amount of coffee: zero, one, two or three cups. The chance that he will not drink any at all is the same as the chance that he will drink three cups and is three times lower than the chance that he will drink exactly one cup. On average, Mr Jones drinks 1.5 cups of coffee per day.

a) Let X denote the number of cups of coffee that Mr Jones drinks on a given day. Find the distribution and the variance of the variable X.

b) A single coffee at a café costs \$3. Mr Jones pays jointly for all the coffees he drinks on a given day, but he only carries ten dollar bills in his wallet. With probability $\frac{2}{3}$ the waiter has only two-dollar coins. In case the waiter can't give back the full change, Mr Jones leaves the remaining dollar as a tip. Find the expected value of the total amount of tips left by Mr Jones over the period of 20 days.

4. Let X be a random variable with a cumulative distribution function:

$$F(t) = \begin{cases} 0 & \text{if } t < 1, \\ \frac{t-1}{6} & \text{if } t \in [1,3), \\ \frac{t-2}{3} + \frac{1}{3} & \text{if } t \in [3,4), \\ 1 & \text{if } t \geq 4, \end{cases}$$

and $Y = X^3$. Find the means of variables X and Y, the median of X and the third quartile of variable Y.

5. Let X be a random variable from an exponential distribution with parameter 4. Calculate the variance of variable $Z = 2 - 5X$. Is variable $Y = \frac{3}{X}$ continuous? If yes, find the density.

6. X is a random variable with a mean equal to $\frac{1}{2}$ and a variance equal to $\frac{1}{15}$. Let Y be a random variable from a distribution with density $f(x) = \left(a(x - \frac{m}{2})^2 + c\right) \mathbf{1}_{[0,m]}(x)$. Find values of parameters a, c and m such that the variables X and Y have the same means and variances.

7. A student has 45 hours she may devote, in any proportion, to prepare for exams from two courses. If t_1 (respectively, t_2) is the time (in hours) devoted to the preparations for the first (respectively, second) course, the probability of passing this course in one attempt amounts to $\frac{t_1}{64}$ (respectively, $\frac{t_2}{49}$). The student may retake the exams multiple times, until they are passed (all trials are independent), but she only prepares once. What should the number of hours devoted to the preparation for the first course exam be equal to, in order to minimize the average number of trials needed to pass exams from both courses?

Probability Calculus Midterm Test. December 3rd, 2019. Version D

In order to obtain the maximum number of points, you need to solve 5 out of the 7 problems below. Each problem must be solved on a separate piece of paper. Please sign each paper with your name and student's number and note the version (A, B, C, D). Each problem will be graded on a scale from 0 to 10 points. Duration: 120 minutes.

1. 18 donuts and 3 muffins were laid out on 7 plates in such a way that there are three cakes on each plate.

a) Calculate the probability that there are three muffins on one of the plates.

b) Assume that it's not the case that there are three muffins on one plate. From a randomly chosen plate we move one randomly chosen cake to a different plate. Calculate the probability that after this move, the three muffins will be on the same plate.

c) Calculate the expected value of the number of donuts on the second plate.

2. There are two types of lottery tickets available in a kiosk: one, where a ticket costs \$2 and the probability of winning amounts to $\frac{1}{150}$ and one, where the ticket costs \$5 and the probability of winning amounts to $\frac{1}{100}$. Mr Smith buys one randomly chosen lottery ticket every day (we assume that tickets from the two lotteries are equally numerous on all days); the choices on different days are independent.

a) Calculate the probability that during three days Mr Smith spent at least \$12, if we know that during this period he won three times.

b) Knowing that during seven days Mr Smith spent \$32 on lottery tickets, calculate the expected value of the number of wins during this period.

c) Using the Poisson theorem, approximate the probability that during 120 days Mr Smith will win at least twice.

3. Each day, Mr Jones drinks a certain amount of coffee: zero, one, two or three cups. The chance that he will not drink any at all is the same as the chance that he will drink three cups and is three times higher than the chance that he will drink exactly one cup. On average, Mr Jones drinks 1.5 cups of coffee per day.

a) Let X denote the number of cups of coffee that Mr Jones drinks on a given day. Find the distribution and the variance of the variable X.

b) A single coffee at a café costs \$3. Mr Jones pays jointly for all the coffees he drinks on a given day, but he only carries ten dollar bills in his wallet. With probability $\frac{1}{4}$ the waiter has only two-dollar coins. In case the waiter can't give back the full change, Mr Jones leaves the remaining dollar as a tip. Find the expected value of the total amount of tips left by Mr Jones over the period of 40 days.

4. Let X be a random variable with a cumulative distribution function:

$$F(t) = \begin{cases} 0 & \text{if } t < 1, \\ \frac{t+1}{6} & \text{if } t \in [1,3), \\ \frac{t-3}{3} + \frac{2}{3} & \text{if } t \in [3,4), \\ 1 & \text{if } t \geq 4, \end{cases}$$

and $Y = X^2$. Find the means of variables X and Y, the median of X and the first quartile of variable Y.

5. Let X be a random variable from an exponential distribution with parameter $\frac{1}{2}$. Calculate the variance of variable $Z = 1 - 2X$. Is variable $Y = \frac{4}{X}$ continuous? If yes, find the density.

6. X is a random variable with a mean equal to $\frac{1}{2}$ and a variance equal to $\frac{1}{20}$. Let Y be a random variable from a distribution with density $f(x) = \left(a(x - \frac{m}{2})^2 + c\right) \mathbf{1}_{[0,m]}(x)$. Find values of parameters a, c and m such that the variables X and Y have the same means and variances.

7. A student has 60 hours she may devote, in any proportion, to prepare for exams from two courses. If t_1 (respectively, t_2) is the time (in hours) devoted to the preparations for the first (respectively, second) course, the probability of passing this course in one attempt amounts to $\frac{t_1}{144}$ (respectively, $\frac{t_2}{64}$). The student may retake the exams multiple times, until they are passed (all trials are independent), but she only prepares once. What should the number of hours devoted to the preparation for the first course exam be equal to, in order to minimize the average number of trials needed to pass exams from both courses?