Probability Calculus Midterm Test
December 2nd, 2016
Version A

Please choose 5 out of the 7 problems below and solve each one on a separate piece of paper. Please sign each paper with your name and student's number. Each problem will be graded on a scale from 0 to 10 points. Duration: 120 minutes.

1. Alice, her two female friends and four male friends sit down randomly around a circular table.

a) What is the probability that no woman will sit next to another woman?

b) Let us assume that there aren't any women sitting next to each other. Alice decides to swap places with the male friend sitting to her left. Calculate the probability that after this change, Alice will sit next to one of her female friends.

c) Calculate the expected value for the number of male friends sitting next to Alice.

2. Mr. Smith commutes by car, and his work starts at 8 AM. His road to work follows three streets. Mr Smith leaves home at 7:30, and in normal traffic conditions reaches his office after 15 minutes. It is possible, however, that on at least one of the three streets traffic lights will experience a failure; a traffic lights failure on a given street prolongs the trip by 10 minutes. Failures on different streets happen independently, and the probability of a single failure is equal to $1/10$. Based on the Poisson Theorem, approximate the probability that during 100 subsequent working days, Mr. Smith will be late to work at least three times. Assess the approximation error.

3. Mr. Smith decides to invest in one of two funds (F_1, or F_2); the choice of the fund is random (each fund has equal chances). The probability that the investment will bring a profit within one year amounts to 97% for F_1 and 98% for F_2.

a) Calculate the probability that after a year from the initial investment, the capital of Mr. Smith will grow.

b) Let us assume that after a year, Mr. Smith's capital grew. Encouraged by this outcome, he decides to invest in the same fund for five additional years. Calculate the probability that during at least two out of the five investment years, the fund will bring profits (for particular years). We assume that the fund outcomes in particular years are independent, and that the probabilities of gains do not change with time.

4. Let X be a random variable from a uniform distribution over $[1, 2]$.

a) Find the variance of $Y = \frac{1}{3+X}$.

b) Find the cumulative distribution function of Y. Is this random variable continuous? Justify your answer.

5. Let X be a random variable with a cumulative distribution function equal to

$$F(t) = \begin{cases} 0 & \text{if } t < 2, \\ \frac{1}{2}t - 1 & \text{if } 2 \le t < 3, \\ \frac{3}{4} & \text{if } 3 \le t < 5, \\ 1 & \text{if } t \ge 5. \end{cases}$$

Calculate the quantile of rank $\frac{1}{5}$ for X and the conditional probability $\mathbb{P}(2^X > 8 | X \in [2, 4])$.

6. Let X be a random variable with density $g(x) = cx\mathbb{1}_{[1,3]}(x)$, and let the distribution of random variable Y be defined by the equalities $\mathbb{P}(Y = k) = ak$, $k \in \{1, 3, 4\}$. Calculate a, c and decide which of the random variables has a larger variance.

7. The duration of a single song (in minutes) has an exponential distribution with parameter $1/3$. Should a song last more than three minutes, when aired it will be cropped to three minutes. Find the expected value of the total time of airing 20 different songs.

Probability Calculus Midterm Test
December 2nd, 2016
Version B

Please choose 5 out of the 7 problems below and solve each one on a separate piece of paper. Please sign each paper with your name and student's number. Each problem will be graded on a scale from 0 to 10 points. Duration: 120 minutes.

1. Alice, her three female friends and five male friends sit down randomly around a circular table.

a) What is the probability that no woman will sit next to another woman?

b) Let us assume that there aren't any women sitting next to each other. Alice decides to swap places with the male friend sitting to her left. Calculate the probability that after this change, Alice will sit next to one of her female friends.

c) Calculate the expected value for the number of male friends sitting next to Alice.

2. Mr. Smith commutes by car, and his work starts at 9 AM. His road to work follows three streets. Mr Smith leaves home at 8:30, and in normal traffic conditions reaches his office after 15 minutes. It is possible, however, that on at least one of the three streets traffic lights will experience a failure; a traffic lights failure on a given street prolongs the trip by 10 minutes. Failures on different streets happen independently, and the probability of a single failure is equal to $1/20$. Based on the Poisson Theorem, approximate the probability that during 200 subsequent working days, Mr. Smith will be late to work at least four times. Assess the approximation error.

3. Mr. Smith decides to invest in one of two funds (F_1, or F_2); the choice of the fund is random (each fund has equal chances). The probability that the investment will bring a profit within one year amounts to 97% for F_1 and 99% for F_2.

a) Calculate the probability that after a year from the initial investment, the capital of Mr. Smith will grow.

b) Let us assume that after a year, Mr. Smith's capital grew. Encouraged by this outcome, he decides to invest in the same fund for five additional years. Calculate the probability that during at least two out of the five investment years, the fund will bring profits (for particular years). We assume that the fund outcomes in particular years are independent, and that the probabilities of gains do not change with time.

4. Let X be a random variable from a uniform distribution over $[2, 3]$.

a) Find the variance of $Y = \frac{1}{1+X}$.

b) Find the cumulative distribution function of Y. Is this random variable continuous? Justify your answer.

5. Let X be a random variable with a cumulative distribution function equal to

$$F(t) = \begin{cases} 0 & \text{if } t < 3, \\ \frac{1}{3}t - 1 & \text{if } 3 \leq t < 4, \\ \frac{2}{3} & \text{if } 4 \leq t < 6, \\ 1 & \text{if } t \geq 6. \end{cases}$$

Calculate the quantile of rank $\frac{1}{4}$ for X and the conditional probability $\mathbb{P}(2^X > 16 | X \in [3, 5])$.

6. Let X be a random variable with density $g(x) = cx\mathbb{1}_{[0,4]}(x)$, and let the distribution of random variable Y be defined by the equalities $\mathbb{P}(Y = k) = ak$, $k \in \{1, 2, 3\}$. Calculate a, c and decide which of the random variables has a larger variance.

7. The duration of a single song (in minutes) has an exponential distribution with parameter $1/4$. Should a song last more than three minutes, when aired it will be cropped to three minutes. Find the expected value of the total time of airing 30 different songs.

Probability Calculus Midterm Test
December 2nd, 2016
Version C

Please choose 5 out of the 7 problems below and solve each one on a separate piece of paper. Please sign each paper with your name and student's number. Each problem will be graded on a scale from 0 to 10 points. Duration: 120 minutes.

1. Alice, her two female friends and four male friends sit down randomly around a circular table.

a) What is the probability that no woman will sit next to another woman?

b) Let us assume that there aren't any women sitting next to each other. Alice decides to swap places with the male friend sitting to her left. Calculate the probability that after this change, Alice will still sit next to two male friends.

c) Calculate the expected value for the number of female friends sitting next to Alice.

2. Mr. Smith commutes by car, and his work starts at 8 AM. His road to work follows three streets. Mr Smith leaves home at 7:00, and in normal traffic conditions reaches his office after half an hour. It is possible, however, that on at least one of the three streets traffic lights will experience a failure; a traffic lights failure on a given street prolongs the trip by 20 minutes. Failures on different streets happen independently, and the probability of a single failure is equal to $1/10$. Based on the Poisson Theorem, approximate the probability that during 200 subsequent working days, Mr. Smith will be late to work at least three times. Assess the approximation error.

3. Mr. Smith decides to invest in one of two funds (F_1, or F_2); the choice of the fund is random (each fund has equal chances). The probability that the investment will bring a profit within one year amounts to 98% for F_1 and 99% for F_2.

a) Calculate the probability that after a year from the initial investment, the capital of Mr. Smith will grow.

b) Let us assume that after a year, Mr. Smith's capital grew. Encouraged by this outcome, he decides to invest in the same fund for six additional years. Calculate the probability that during at least two out of the six investment years, the fund will bring profits (for particular years). We assume that the fund outcomes in particular years are independent, and that the probabilities of gains do not change with time.

4. Let X be a random variable from a uniform distribution over $[1,2]$.

a) Find the variance of $Y = \frac{1}{2+X}$.

b) Find the cumulative distribution function of Y. Is this random variable continuous? Justify your answer.

5. Let X be a random variable with a cumulative distribution function equal to

$$F(t) = \begin{cases} 0 & \text{if } t < 2, \\ \frac{1}{2}t - 1 & \text{if } 2 \le t < 3, \\ \frac{2}{3} & \text{if } 3 \le t < 6, \\ 1 & \text{if } t \ge 6. \end{cases}$$

Calculate the quantile of rank $\frac{1}{3}$ for X and the conditional probability $\mathbb{P}(3^X > 27 | X \in [2,5])$.

6. Let X be a random variable with density $g(x) = cx\mathbb{1}_{[2,4]}(x)$, and let the distribution of random variable Y be defined by the equalities $\mathbb{P}(Y = k) = ak$, $k \in \{1,\ 2,\ 4\}$. Calculate a, c and decide which of the random variables has a larger variance.

7. The duration of a single song (in minutes) has an exponential distribution with parameter $1/3$. Should a song last more than four minutes, when aired it will be cropped to four minutes. Find the expected value of the total time of airing 20 different songs.

Probability Calculus Midterm Test
December 2nd, 2016
Version D

Please choose 5 out of the 7 problems below and solve each one on a separate piece of paper. Please sign each paper with your name and student's number. Each problem will be graded on a scale from 0 to 10 points. Duration: 120 minutes.

1. Alice, her three female friends and five male friends sit down randomly around a circular table.

a) What is the probability that no woman will sit next to another woman?

b) Let us assume that there aren't any women sitting next to each other. Alice decides to swap places with the male friend sitting to her left. Calculate the probability that after this change, Alice will still sit next to two male friends.

c) Calculate the expected value for the number of female friends sitting next to Alice.

2. Mr. Smith commutes by car, and his work starts at 9 AM. His road to work follows three streets. Mr Smith leaves home at 8:00, and in normal traffic conditions reaches his office after half an hour. It is possible, however, that on at least one of the three streets traffic lights will experience a failure; a traffic lights failure on a given street prolongs the trip by 20 minutes. Failures on different streets happen independently, and the probability of a single failure is equal to $1/20$. Based on the Poisson Theorem, approximate the probability that during 100 subsequent working days, Mr. Smith will be late to work at least three times. Assess the approximation error.

3. Mr. Smith decides to invest in one of two funds (F_1, or F_2); the choice of the fund is random (each fund has equal chances). The probability that the investment will bring a profit within one year amounts to 96% for F_1 and 98% for F_2.

a) Calculate the probability that after a year from the initial investment, the capital of Mr. Smith will grow.

b) Let us assume that after a year, Mr. Smith's capital grew. Encouraged by this outcome, he decides to invest in the same fund for six additional years. Calculate the probability that during at least two out of the six investment years, the fund will bring profits (for particular years). We assume that the fund outcomes in particular years are independent, and that the probabilities of gains do not change with time.

4. Let X be a random variable from a uniform distribution over $[2,3]$.

a) Find the variance of $Y = \frac{1}{2+X}$.

b) Find the cumulative distribution function of Y. Is this random variable continuous? Justify your answer.

5. Let X be a random variable with a cumulative distribution function equal to

$$F(t) = \begin{cases} 0 & \text{if } t < 3, \\ \frac{1}{3}t - 1 & \text{if } 3 \leq t < 5, \\ \frac{3}{4} & \text{if } 5 \leq t < 7, \\ 1 & \text{if } t \geq 7. \end{cases}$$

Calculate the quantile of rank $\frac{1}{4}$ for X and the conditional probability $\mathbb{P}(2^X > 16 | X \in [3,6])$.

6. Let X be a random variable with density $g(x) = cx\mathbb{1}_{[0,2]}(x)$, and let the distribution of random variable Y be defined by the equalities $\mathbb{P}(Y = k) = ak$, $k \in \{2,\ 3,\ 4\}$. Calculate a, c and decide which of the random variables has a larger variance.

7. The duration of a single song (in minutes) has an exponential distribution with parameter $1/4$. Should a song last more than four minutes, when aired it will be cropped to four minutes. Find the expected value of the total time of airing 30 different songs.