

**Probability Calculus Midterm Test**  
**December 4th, 2015**  
**Version B**

Please choose 5 out of the 6 problems below and solve each one on a separate piece of paper. Each problem will be graded on a scale from 0 to 10 points. Duration: 120 minutes.

- Bob has 2 dollars, and Al has 1 dollar. A drink in a vending machine costs 3 dollars, but Bob and Al have different tastes. They have decided to apply the following rule: they (repeatedly) toss a coin: if heads appear, Al gives a dollar to Bob; if tails appear, Bob gives a dollar to Al. Who is the first to have 3 dollars wins and decides on the taste of the drink. There is not much time left until the end of the coffee break – for at most 6 tosses. Calculate the probability that until the end of the coffee break (a) Al wins (b) Bob wins, (c) Al wins, provided that the game is decided. ★ Calculate the probability that Al will be the winner, if the game does not have a time limit.
- A shop manager tests three types of discount strategies: 20% randomly chosen clients get the first type, 30% get the second one, and the remaining clients get the third type. It turns out that clients from the first group purchase again with probability  $\frac{1}{3}$ , from the second group – with probability  $\frac{1}{5}$ , and from the third program – with probability  $\frac{1}{4}$ .
  - A client purchased again. What is the probability that he was under the first discount strategy?
  - The discount strategy is randomly assigned again (independently of the previous assignment) if a client returns to the store. Mr X returned twice. What is the chance that he was assigned to the first program twice?
- A company deals with telephone calls from 640 clients per day. The clients may call at any, randomly chosen, minute during the twenty-four hours. A client's telephone calls last on average 4.5 minutes per day. What is the chance (approximately) that if two workers service the telephone line, no client will have to wait for a free line at 9 : 00 AM? How many workers should be employed in order to assure that with probability at most 0.05 (approximately) someone will have to wait for a free line?  
*Hint:*  $e^{-4.5/320} \approx 0.99$      $e^{-1} \approx 0.37$      $e^{-2} \approx 0.14$      $e^{-4.5} \approx 0.01$
- The price  $P$  of share on the stock exchange is a random variable from a uniform distribution over  $[18, 24]$ . An investor has the choice between two instruments: one (A), for which the return (dependent on the price  $P$ ) equals to  $\frac{P-21}{3}$  and a different one (B), for which the return is equal to  $\frac{(P-21)^5}{243}$ . (a) Find the distribution of the returns for instruments A and B. (b) Which of the two instruments (A or B) would be chosen by a risk-seeking investor who decides based on the variance of the distribution?
- Let us assume that the random variable  $X$  describing the life duration in a population has a distribution such that

$$\mathbf{P}(X > t) = \begin{cases} \exp(-(t - 95)) & \text{for } t > 120 \\ \exp(-\frac{t-70}{2}) & \text{for } 80 < t \leq 120 \\ \exp(-\frac{t-55}{5}) & \text{for } 60 < t \leq 80 \\ \exp(-\frac{t}{60}) & \text{for } 0 \leq t \leq 60 \end{cases} .$$

- Is  $X$  a continuous random variable? Justify your answer.
  - Find the mean life duration in the population.
  - Calculate the median of the life duration in the population. *Hint: see the hint for problem 3*
- The monthly number of transactions in which a real-estate agent takes part in is a random variable from a Binomial distribution with parameters 4 and  $\frac{1}{4}$  if the agent has not been trained, a Binomial distribution with parameters 6 and  $\frac{1}{4}$  for an agent who has undergone standard training, and the square of the number of transactions of an untrained agent for an agent who has undergone extra training. (a) Find the mean and the variance of the monthly number of transactions of an agent who has had extra training. (b) What is more profitable from the point of view of an agency which has hired four untrained workers, in terms of the average monthly number of transactions: provide standard training to all new workers or provide extra training for two workers?