

Probability Calculus Midterm Test
December 8th 2012

1. Events A and B are independent and $P(A) = x$, $P(B) = 1 - x$ and $P(A' \cap B) = \frac{1}{4}$. Find x . (4 pts)
2. In a population there are approximately 94% cautious drivers, and 6% road hogs. A cautious driver causes an accident with probability 5% (per calendar year), and a road hog – with probability 20%. A new client approaches an insurance company and states that he has not caused an accident within the last year. What is the probability that this client is a cautious driver? (3 pts) What is the probability that he will not commit an accident within the next five years? (3 pts)
3. The banking system erroneously handles the data of a transfer recipient in, on average, 2 out of 10^7 bank transfers. Based on the Poisson approximation, assess the probability that out of 10^6 transfers that were entered in a given month, at least three transfers were handled erroneously (the recipient was a well-known politician). (4 pts) Assess the approximation error. (1 pt)
4. Let X be a continuous random variable with density $f(x) = \begin{cases} c \cos x & 0 < x < \frac{\pi}{2} \\ 0 & \text{otherwise} \end{cases}$ Calculate
 - (a) c , (1 pts)
 - (b) $P(\frac{1}{2} < x \leq 2)$, (2 pts)
 - (c) the cumulative distribution function of X . (3 pts)
5. Let X be a random variable with a uniform distribution on $[0, 3]$. Find the distribution of the random variable $Y = \min\{X, X^2\}$, (4 pts) and the median of Y (1 pt). Is the distribution of Y continuous? discrete? Justify your answer. (2 pts)
6. There are 5 black balls and 5 white balls in a box. We conduct the following experiment: we draw a ball from the box. If it is black, we put it back in the box. If it is white, we put it aside. We repeat the experiment until all the white balls are taken out of the box. Calculate the average number of draws. (6 pts)
7. Let X be a random variable from an exponential distribution with parameter 1, and let $Y = X^2 + 1$. Calculate the expected value (3 pts) and the variance (3 pts) of Y .