Probability Calculus Final Exam Retake - 07.03.2018
group A

Each problem should be solved on a separate piece of paper, you should return all 6. Each problem will be graded on a scale from 0 to 10 points. The exam result is the sum of points obtained for the 5 problems with highest grades. Please sign each piece of paper with your name and student's number and the group sign (A, B, ...). When dealing with the CDF of the standard normal distribution, please use the Φ notation, and not values from tables. Duration: 120 minutes.

1. Two individuals take part in a lottery game. Each of them chooses 6 numbers from the set $\{1, 2, \ldots, 49\}$, then the chosen sets of numbers are compared.

a) Calculate the probability of the event that exactly one number appeared twice (i.e., was chosen by both individuals).

b) Find the expected value of the number of repeated numbers (i.e., numbers chosen by both individuals).

2. The monthly number of accidents on a given crossing is a random variable X such that $\mathbb{P}(X = 0) = \mathbb{P}(X = 1) = \mathbb{P}(X = 3) = \mathbb{P}(X = 4) = \frac{1}{6}$, $\mathbb{P}(X = 2) = \frac{1}{3}$. Each accident, independently of others, may either be serious or light (which happens with probabilities $\frac{1}{5}$ and $\frac{4}{5}$, respectively). Let Y denote the number of serious accidents in February. Calculate $\mathbb{P}(X = 3|Y = 2)$ and $\mathbb{E}(Y|X)$.

3. Let (X, Y) be a two-dimensional normal random variable. We know that variables $X + Y$ and Y are independent, with means equal to -1 and variances equal to 2. Calculate $\mathbb{E}XY$ and find the distribution of random variable $2X + Y + 1$.

4. A two-dimensional random variable (X, Y) may be described by a density function $g(x, y) = 72x^{-4}y^2\mathbb{1}_{\{x\geq 2,\, 0\leq y\leq 1\}}$. Calculate $\mathbb{P}(XY > 1)$ and $\mathrm{Var}\,(XY^{-1} + 1)$.

5. Mr. Smith invests in currencies. At the beginning of each month, he buys 100 or 200 crowns (each possibility has the same probability).

a) Approximate the probability that during 100 months, Mr. Smith will buy at most 15200 crowns.

b) In the middle of each month, Mr. Smith tosses a coin, on which heads appear with probability 1/3. If heads appear, Mr. Smith does not do anything more; if tails appear, he allots half of the amount of crowns bought that month to buy gold coins. Approximate the probability, that during 150 months Mr. Smith will allot less than 8 thousand crowns to buy gold coins.

6. A flock of doves travels between four cities: M_1, M_2, M_3, M_4, which are located in this order on the banks of the Vistula river. Each evening, the flock decides to either stay in the city where they are, or travel to one of the adjacent cities (each option has the same probability; the flight between cities takes a whole night). The flock spends the first day in city M_1.

a) What is more probable: that on the morning of the third day, the flock will be in the city M_1, or that on the morning of the third day, it will be in the city M_3?

b) After how many days, on average, will the flock visit city M_4 for the first time?

c) Approximate the probability that after 100 days, the flock will be in city M_4 in the morning.

Probability Calculus Final Exam Retake - 07.03.2018
group B

Each problem should be solved on a separate piece of paper, you should return all 6. Each problem will be graded on a scale from 0 to 10 points. The exam result is the sum of points obtained for the 5 problems with highest grades. Please sign each piece of paper with your name and student's number and the group sign (A, B, ...). When dealing with the CDF of the standard normal distribution, please use the Φ notation, and not values from tables. Duration: 120 minutes.

1. Two individuals take part in a lottery game. Each of them chooses 5 numbers from the set $\{1, 2, \dots, 49\}$, then the chosen sets of numbers are compared.

a) Calculate the probability of the event that exactly one number appeared twice (i.e., was chosen by both individuals).

b) Find the expected value of the number of repeated numbers (i.e., numbers chosen by both individuals).

2. The monthly number of accidents on a given crossing is a random variable X such that $\mathbb{P}(X = 0) = \mathbb{P}(X = 1) = \mathbb{P}(X = 3) = \mathbb{P}(X = 4) = \frac{1}{7}$, $\mathbb{P}(X = 2) = \frac{3}{7}$. Each accident, independently of others, may either be serious or light (which happens with probabilities $\frac{1}{6}$ and $\frac{5}{6}$, respectively). Let Y denote the number of serious accidents in February. Calculate $\mathbb{P}(X = 4|Y = 2)$ and $\mathbb{E}(Y|X)$.

3. Let (X, Y) be a two-dimensional normal random variable. We know that variables $X + Y$ and Y are independent, with means equal to 1 and variances equal to 3. Calculate $\mathbb{E}XY$ and find the distribution of random variable $X + 2Y - 1$.

4. A two-dimensional random variable (X, Y) may be described by a density function $g(x, y) = 72x^2y^{-4}\mathbb{1}_{\{0\leq x\leq 1,\, y\geq 2\}}$. Calculate $\mathbb{P}(XY > 1)$ and $\mathrm{Var}\,(X^{-1}Y + 2)$.

5. Mr. Smith invests in currencies. At the beginning of each month, he buys 100 or 300 crowns (each possibility has the same probability).

a) Approximate the probability that during 100 months, Mr. Smith will buy at most 20200 crowns.

b) In the middle of each month, Mr. Smith tosses a coin, on which heads appear with probability 3/4. If heads appear, Mr. Smith does not do anything more; if tails appear, he allots half of the amount of crowns bought that month to buy gold coins. Approximate the probability, that during 100 months Mr. Smith will allot less than 3 thousand crowns to buy gold coins.

6. A flock of doves travels between four cities: M_1, M_2, M_3, M_4, which are located in this order on the banks of the Vistula river. Each evening, the flock decides to either stay in the city where they are, or travel to one of the adjacent cities (each option has the same probability; the flight between cities takes a whole night). The flock spends the first day in city M_4.

a) What is more probable: that on the morning of the third day, the flock will be in the city M_4, or that on the morning of the third day, it will be in the city M_2?

b) After how many days, on average, will the flock visit city M_1 for the first time?

c) Approximate the probability that after 100 days, the flock will be in city M_2 in the morning.

Probability Calculus Final Exam Retake - 07.03.2018
group C

Each problem should be solved on a separate piece of paper, you should return all 6. Each problem will be graded on a scale from 0 to 10 points. The exam result is the sum of points obtained for the 5 problems with highest grades. Please sign each piece of paper with your name and student's number and the group sign (A, B, ...). When dealing with the CDF of the standard normal distribution, please use the Φ notation, and not values from tables. Duration: 120 minutes.

1. Two individuals take part in a lottery game. Each of them chooses 7 numbers from the set $\{1, 2, \ldots, 49\}$, then the chosen sets of numbers are compared.

a) Calculate the probability of the event that exactly one number appeared twice (i.e., was chosen by both individuals).

b) Find the expected value of the number of repeated numbers (i.e., numbers chosen by both individuals).

2. The monthly number of accidents on a given crossing is a random variable X such that $\mathbb{P}(X = 0) = \mathbb{P}(X = 1) = \mathbb{P}(X = 3) = \mathbb{P}(X = 4) = \frac{1}{8}$, $\mathbb{P}(X = 2) = \frac{1}{2}$. Each accident, independently of others, may either be serious or light (which happens with probabilities $\frac{1}{3}$ and $\frac{2}{3}$, respectively). Let Y denote the number of serious accidents in February. Calculate $\mathbb{P}(X = 4|Y = 3)$ and $\mathbb{E}(Y|X)$.

3. Let (X, Y) be a two-dimensional normal random variable. We know that variables $X - Y$ and Y are independent, with means equal to -2 and variances equal to 2. Calculate $\mathbb{E}XY$ and find the distribution of random variable $3X + Y - 1$.

4. A two-dimensional random variable (X, Y) may be described by a density function $g(x, y) = \dfrac{9}{8}x^{-4}y^2\mathbb{1}_{\{x\geq 1,\, 0\leq y\leq 2\}}$. Calculate $\mathbb{P}(XY > 1)$ and $\mathrm{Var}\,(2XY^{-1} - 3)$.

5. Mr. Smith invests in currencies. At the beginning of each month, he buys 100 or 300 crowns (each possibility has the same probability).

a) Approximate the probability that during 100 months, Mr. Smith will buy more than 19800 crowns.

b) In the middle of each month, Mr. Smith tosses a coin, on which heads appear with probability 1/4. If heads appear, Mr. Smith does not do anything more; if tails appear, he allots half of the amount of crowns bought that month to buy gold coins. Approximate the probability, that during 150 months Mr. Smith will allot more than 12 thousand crowns to buy gold coins.

6. A flock of doves travels between four cities: M_1, M_2, M_3, M_4, which are located in this order on the banks of the Vistula river. Each evening, the flock decides to either stay in the city where they are, or travel to one of the adjacent cities (each option has the same probability; the flight between cities takes a whole night). The flock spends the first day in city M_2.

a) What is more probable: that on the morning of the third day, the flock will be in the city M_1, or that on the morning of the third day, it will be in the city M_3?

b) After how many days, on average, will the flock visit city M_4 for the first time?

c) Approximate the probability that after 100 days, the flock will be in city M_3 in the morning.