**Probability Calculus Resit Exam - 02.03.2016**
**group A**

*Each problem should be solved on a separate piece of paper, you should return all 6. Solving 5 out of 6 problems correctly will give you the maximum number of points for the exam. Each problem will be graded on a scale from 0 to 10 points. Please sign each piece of paper with your name and student's number. Duration: 120 minutes.*

1. The clients of bank B may either be economical or overspending (the latter in 80% cases). An economical client puts aside at least \$1000 on a savings account in a given month with probability 0.6, while an overspending client – with probability 0.2. We assume that the amounts saved in subsequent months by a given client are independent. Based on account history, the bank would like to offer his economical clients a special offer.
   (a) Calculate the probability that a randomly chosen client of the bank puts aside at least \$1000 at least 3 times within half a year.
   (b) A client put aside \$1000 twice during the last three months. What is the probability that the client is economical?

2. Assume that the time a client spends at a cash desk is a random variable $X$ from a distribution with density
$$f(x) = cx^2 1_{(0,3)}(x).$$
   Assume that instead of observing $X$, we observe $Y = \min\{X, 1\}$.
   (a) Find the constant $c$.
   (b) Find the distribution of $Y$. Does $Y$ have a continuous distribution?
   (c) Calculate $\mathbb{E}Y$, $\mathrm{Var}Y$.
   (d) Find $\mathbb{E}(Y|X)$.

3. Let $(X, Y)$ be a random vector with density $g(x, y) = \frac{1}{2\pi} e^{(x-1)^2/2+(y-3)^2/2}$.
   (a) Are $X$ and $Y$ independent? Justify your answer.
   (b) Find the correlation coefficient of $X + Y$ and $X - Y$.
   (c) What is the distribution of the random variable $Z = (X - 1)^2 + (Y - 3)^2$?

4. Assume that the joint density of the random vector $(X, Y)$ is equal to
$$f(x, y) = \frac{1}{16} 1_{\{(x,y):|x|+2|y|\leqslant 4\}}.$$
   (a) Calculate the conditional density $f_{X|Y}(x|y)$.
   (b) Find $\mathbb{E}(X|Y)$.
   (c) Are $X, Y$ independent? Justify your answer!
   (d) Are $X, Y$ uncorrelated?

5. Two airlines operate flights between Warsaw and Stockholm, where 400 passengers travel daily. The passengers choose the carrier randomly and independently of each other. Approximate the probability that if the "Air X" airplane has 210 seats, on March 2nd the flight will be overbooked. How many seats should the aircraft have in order to assure that with probability 0.994 all passengers fly with the airline of their choice?

6. Mr X transfers between home, work, shopping center and bar. If Mr X is at home, with probability 1/2 he goes to work, and with probabilities 1/4 he goes shopping or to a bar. From work, Mr X moves home or goes shopping with probabilities 1/2 each. From a shopping center, Mr X transfers home with probability 1/2, and with probabilities 1/4 shopping or to a bar. When at a bar, he goes home with probability 3/4 and with probability 1/4 to a bar again. Let us assume that initially Mr X is at home.
   (a) What is the probability that after 10000 moves he will be at home again?
   (b) How many times on average will he transfer until he arrives at home again for the first time?
   (c) What is the probability that Mr X will arrive at work before he arrives at a bar?

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$\Phi(0) = 0.5$, $\Phi(1) \approx 0.841$, $\Phi(1.5) \approx 0.933$, $\Phi(2) \approx 0.977$, $\Phi(2.5) \approx 0.994$, $\Phi(3) \approx 0.9987$, $\Phi(4) \approx 0.99997$