**Probability Calculus Retake Exam - 5.03.2015**
**group A**

*Each problem should be solved on a separate piece of paper, you should return all 6. Solving 5 out of 6 problems correctly will give you the maximum number of points for the exam. Each problem will be graded on a scale from 0 to 10 points. Please sign each piece of paper with your name and student's number. Duration: 120 minutes.*

1. There are two boxes; in the first one, there are 5 normal coins, and in the second one, there are 2 coins — one normal, and one with heads on both sides. We randomly draw a box and then a coin from the box, and we toss the drawn coin twice. Calculate the probability that
   (a) heads will appear at least once;
   (b) we drew the special coin, given that there were heads in both tosses.

2. 2/3 of workers in a corporation are rank-and-file, and the remaining 1/3 are CEOs. The monthly earnings of a random ordinary worker have a uniform distribution over $[1000, 2000]$, and the monthly earnings of a random CEO have a distribution given by the following CDF:
$$F(t) = \begin{cases} 0; & t < 2000 \\ 1 - (\frac{2000}{t})^2; & t \geqslant 2000. \end{cases}$$
   Let $X$ denote the monthly earnings of a random worker of the corporation.
   (a) Find the CDF and the median of $X$.
   (b) Calculate $\mathbb{E}X$.
   (c) Determine whether $X$ has a finite variance. Justify your answer!

3. Let $(X, Y)$ be a random vector with density $g(x, y) = C\frac{1}{x^4}1_{\{x \geqslant 1\}}1_{\{x-1 \leqslant y \leqslant x+1\}}$.
   (a) Find the constant $C$.
   (b) Find the density of $X$ and $\mathbb{P}(X > 2)$.
   (c) Calculate $\mathbb{E}X$ and $\mathrm{Cov}(X, Y)$.

4. Let $D$ denote the income of Mr X, a salesman distributing cosmetics of company A. Assume this income depends on the sales of Mr X (25% of the worth of the sold products) and on the sales of salesmen encouraged by him to become distributors (10% of the worth of sold products). Assume further that the number of individuals encouraged by Mr X (denoted by $K$) is a random variable from a binomial distribution with parameters 6 and $\frac{2}{3}$, and that given $k$, the worth of monthly sales of $k$ encouraged distributors is a random variable from a uniform distribution over $[0, 1500k]$, while the worth of sales of Mr X is a random variable with mean 2000. Calculate the average monthly earnings of Mr X and $\mathrm{Cov}(K, D)$.

5. The worth of items purchased by clients in a clothing shop are independent random variables from a distribution with mean 100 and standard deviation equal to 100. Approximate the probability that the total income from 400 clients will not exceed 36000. Assume further that upon the first purchase, each client receives a 50% discount coupon for the next purchase. Approximate the probability that the total income from 500 clients (two visits each) will exceed 77500.

6. In an economy the following mechanism of changes in workers' sector of employment may be observed (we assume that each employed individual may change the sector of employment once every half a year): if somebody is employed in agriculture, he will change to services with probability $\frac{1}{4}$ and with probability $\frac{1}{4}$ to manufacturing in the next period; if an individual is employed in manufacturing, he will switch to services with probability $\frac{1}{4}$ in the next period; if a worker is employed in services, in the next period he will switch to agriculture with probability $\frac{1}{10}$; in all other cases, the worker remains employed in the same sector. Calculate:
   (a) the probability that an individual employed in services in the first half of 2015 will be employed in services in the first half of 2016;
   (b) the average time which will pass until a worker employed in agriculture in 2015 will become employed in services;
   (c) the long-term structure of employment in the economy.

---

$\Phi(0) = 0,5$, $\Phi(1) \approx 0,841$, $\Phi(1,5) \approx 0,933$, $\Phi(2) \approx 0,977$, $\Phi(2,5) \approx 0,994$, $\Phi(3) \approx 0,9987$, $\Phi(4) \approx 0,99997$