Fill in the dotted spaces [".........."]. 1 question (•) = 1 point; maximum = 16 points. Only responses in the specified places will be checked, without justifications or calculations. Fill in your responses **after having verified them**; if illegible or larded with corrections and crossings-out, the answers will be treated as wrong. You can use a simple calculator, statistical tables and one a4 sheet of paper with helpful formulas. **Communication with others is not allowed**.

NAME: .... student's number ....

1. A restaurant manager wishes to determine whether the way in which the restaurant menu is presented to the clients has any impact on the number of dishes ordered and the total billed amount in a newly opened venue. Therefore, she randomly assigns clients to two groups: the first group is presented with a written paper version of the menu, while the second group is informed about the available dishes orally by the waiter. The average number of dishes ordered by 16 clients from the first group was equal to 2.5, while for the second group of 16 clients, the average was 2.3. From previous experience, the manager knew that the variance of the number of dishes ordered amounts to 2 for written menus and 2.5 for menus presented orally. Meanwhile, the average amount spent by a client from the first group was equal to \$40 and for the second group – \$50. The manager also calculated the standard deviations (based on the unbiased estimators of the variance) for the amounts spent, and they were equal for the two groups (and amounted to \$10).

   - Assuming that the number of dishes ordered follows a normal distribution, verify the hypothesis that the two groups of clients do not differ in terms of the number of dishes ordered. The value of the appropriate test statistic is equal to ...., the critical value for the 5% significance level is equal to ...., so we REJECT /DO NOT HAVE GROUNDS TO REJECT the null hypothesis (underline the appropriate).
   - Assuming that the amounts spent follow a normal distribution, verify the hypothesis that the clients in the second group spend on average at least as much as the clients in the first group. The value of the appropriate test statistic is equal to ...., the critical value for the 5% significance level is equal to ...., so we REJECT /DO NOT HAVE GROUNDS TO REJECT the null hypothesis (underline the appropriate).

2. We continue the analysis of data from the previous question.

   - We wish to construct 95% confidence intervals for the amounts spent by the two groups of clients. The realization of the confidence interval for the first group of clients is equal to .... The interval for the second group is WIDER/NARROWER /OF THE SAME LENGTH (underline the appropriate).
   - If the realization of a 95% confidence interval for the mean for the first group includes the average calculated for the second group, this means that the averages in the two groups ARE EQUAL /ARE NOT EQUAL /MAY OR MAY NOT BE EQUAL (underline the appropriate).

3. As a continuation of the menu presentation experiment, the manager studied the fraction of clients who order the most expensive dish. In a sample of 100 clients, there were 40 who ordered it when the menu was presented to them orally by the waiter.

   - The realization of the 90% confidence interval for the fraction ordering the most expensive dish is equal to ....

- Based on the sample, we verify the hypothesis that the fraction of clients who order the most expensive dish is at least 30%. The value of the test statistic is equal to ...., the corresponding p-value is ...., so for a 5% significance level we REJECT /DO NOT HAVE GROUNDS TO REJECT the null hypothesis (underline the appropriate).

4. Let $X_1, X_2, \ldots, X_n$ be random variables from a distribution with density

$$f(x) = \theta 2^\theta x^{\theta-1}$$

for $x \in (0, \frac{1}{2})$ and 0 otherwise, for an unknown parameter $\theta > 0$.

- The Method of Moments estimator of θ, based on the mean, is equal to

....

- The Maximum Likelihood estimator of θ is equal to

....

5. We analyze the time spent by groups of clients dining in a restaurant relative to the type of music playing in the background and its loudness, based on observations of the behavior of clients on four different days, when the settings were changed. The collected data are summarized in the table below.

Day Music settings	1 Silent pop	2 Loud pop	3 Silent classic	4 Loud classic
Average time spent (hours)	1.25	1	1.5	1
Variance of time spent (biased estimator)	0.25	0.25	0.36	0.36
Number of clients	40	30	40	40

We assume that the time spent dining follows a normal distribution and clients do not return to the restaurant on subsequent days. We conduct an analysis of variance test for the four days to verify the null hypothesis that the music settings do not impact the average time spent dining.

- The sum of squares between groups (SSB) is equal to ...., and the sum of squares within groups (SSW) is equal to .... The value of the test statistic is equal to ....
- The critical region for a 1% significance level is equal to ...., so for this significance level we REJECT /DO NOT HAVE GROUNDS TO REJECT the null hypothesis (underline the appropriate). Based on this result we can say that THE TYPE /THE VOLUME /NEITHER THE TYPE NOR THE VOLUME /THE TYPE AND THE VOLUME COMBINED have impact on the dining duration (underline the appropriate).

6. Based on a single observation from a Poisson distribution with parameter λ, we verify the null hypothesis that $\lambda = 2$ against the alternative that $\lambda = 3$ using the most powerful test for a significance level equal to 0.02.

- The critical region of the test is equal to ....
- The power of the test is equal to .... Would you like to get 1 point for this problem without doing the calculations? YES /slash NO (underline the appropriate).

Hint. You can use the following approximations in your calculations:

$e^{-1} \approx 0.37; \quad e^{-2} \approx 0.14; \quad e^{-3} \approx 0.05$

7. The tips left to a waiter by restaurant clients on a given week may be summarized by the following table:

Tip amount (in dollars)	0	(0, 5]	(5, 10]	(10,20]
Number of clients	5	20	10	5

Based on these aggregate results, calculate:

- the average value of the tip amount = ...;
  the variance of the tip amount = ...
- the median value of the tip amount= ....;
  the first decile value of the tip amount = ...

8. Let $X_1, X_2, \ldots, X_n$ be random variables from a distribution with density

$$f(x) = \frac{3x^2}{\theta^3}$$

for $x \in (0, \theta)$ and 0 otherwise, for an unknown parameter $\theta > 0$. Let $\hat{\theta} = \frac{3}{2}\bar{X}$ be an estimator of θ.

- The bias of the estimator $\hat{\theta}$ is equal to ....
- The variance of the estimator $\hat{\theta}$ is equal to ...., and the MSE of this estimator is thus equal to ....