Mathematical Statistics Exam, June 2016, set B

Fill in the dotted spaces ["........."]. 1 question (•) = 1 point; maximum = 16 points. Only responses in the specified places will be checked, without justifications or calculations. Fill in your responses **after having verified them**; if illegible or larded with corrections and crossings-out, the answers will be treated as wrong. You can use a calculator, statistical tables and one a4 sheet of paper with helpful formulas. **Communication with others is not allowed**.

NAME: .... student's number ....

Signature ....

1. Swiss frank and euro exchange rates in 20 Warsaw currency exchange bureaus were studied. The sample average exchange rate for the Swiss frank amounted to 4.10, and for the euro: 4.44, with variances (calculated with the unbiased formula) equal to 0.20 and 0.25, respectively. We assume that exchange rates follow normal distributions.

   - Find the 95% confidence interval for the mean exchange rate for the euro.

     ....

     An analogous confidence interval for a confidence level of 90% would be .... times WIDER /SHORTER (underline the appropriate).

   - For a significance level of 0.05 we verify the null hypothesis that the variances for the two currencies are the same, against the alternative that the variance for euro is higher. The value of the appropriate test statistic is equal to

     ....,

     the critical value of the appropriate test is equal to ...., so we should REJECT /DO NOT HAVE GROUNDS TO REJECT the null (underline the appropriate).

2. Deepening the analysis of exchange rates, an additional factor of bureau type was studied. Aggregate results are presented below (variance calculated with the unbiased formula).

Bureau type	count	average euro exchange rate	variance of euro exchange rate
normal	11	4.4	0.4
internet	11	4.3	0.2
bank	6	4.5	0.3

Verify the hypothesis of equality of exchange rates between institutions of different types, with the analysis of variance test for a significance level 0.01.

- The sum of squares within groups is equal to ....,
  the sum of squares between groups is equal to ....,

  so the value of the appropriate test statistic is equal to ....
- The critical value for the test amounts to ...., so we REJECT /DO NOT HAVE GROUNDS TO REJECT (underline the appropriate) the hypothesis of equality of means.

3. Let $X_1, X_2, \ldots, X_n$ be a sample from a distribution with density

$$f_\theta(x) = (\theta + 1)x^\theta \cdot \mathbf{1}_{(0,1)}(x),$$

where $\theta \in (0, 1)$ is an unknown parameter.

- The method of moments estimator for θ, based on the mean, is equal to

  ....
- The Bayesian Most Probable Estimator of θ, assuming an *a priori* uniform distribution on $(0, 1)$, is equal to

  ....

4. Let $X_1, X_2, \ldots, X_n$ be independent random variables from a distribution with density

$$f_\theta(x) = \frac{1}{\theta} e^{-\frac{x}{\theta}} \cdot \mathbf{1}_{(0,\infty)}(x),$$

where $\theta > 0$ is an unknown parameter. Let $\hat{\theta}_1 = \bar{X}$ and $\hat{\theta}_2 = nX_{1:n}$ be estimators of θ.

- The bias of $\hat{\theta}_1$ is equal to ....,

  while the bias of $\hat{\theta}_2$ is equal to ....
- The mean square error of $\hat{\theta}_1$ is equal to ....,

  while for $\hat{\theta}_2$ is equal to ...., so for $n > 1$ the $\hat{\theta}_1$ estimator is MORE EFFICIENT /LESS EFFICIENT /EQUALLY EFFICIENT /WE SHOULD NOT TALK ABOUT EFFICIENCY (underline the appropriate) than $\hat{\theta}_2$.

Hint. If $X_1, X_2, \ldots, X_n$ are independent random variables from an exponential distribution with parameter λ, then $\min\{X_1, X_2, \ldots, X_n\}$ has an exponential distribution with parameter $n\lambda$.

5. Let X be a single observation from a distribution with density

$$f_\theta(x) = \frac{\theta}{2^\theta} x^{\theta-1} \cdot \mathbf{1}_{(0,2)}(x),$$

where $\theta > 0$ is an unknown parameter. We test the null hypothesis that $\theta = 3$ against the alternative that $\theta = 1$ with the most powerful test for a significance level $\alpha = 0.05$.

- The critical region of the test is ....
- The power of the test for the alternative hypothesis is ....

6. Interest rates (in %) for fixed term deposits were analyzed. The aggregate results for numbers of offers falling into specified ranges are presented below:

	(1; 1.1)	[1.1; 1.2)	(1.2; 1.3]	[1.3; 1.4)
Number of 3-month deposits	30	30	20	20
Number of 6-month deposits	24	24	26	26

   - The average interest rate for 3-month deposits is equal to ...., and the variance is equal to .... For 6-month deposits, the average is ...., and the variance is ....
   - The coefficients of variation for 3- and 6- month deposits are equal to .... and ...., respectively, which means that the interest rate for 6-month deposits has HIGHER /LOWER /EQUAL variability (underline the appropriate).

7. We continue the analysis of exchange rates data presented in the problem above. We verify the hypothesis that the distribution of interest rates does not depend on the type of deposit, with a chi-squared test at a significance level of 0.05.
   - The appropriate test statistic has .... degrees of freedom, and the value of the statistic for the data above is equal to ....
   - The critical region of the appropriate test is equal to

     ....,

     so we REJECT /DO NOT HAVE GROUNDS TO REJECT (underline the appropriate) the null.

8. Further analyzing the interest rate data presented above, we look at the distribution of interest rates for 6-month deposits.
   - We verify the null that the fraction of deposits with interest rates smaller than 1.2% is equal to 0.5 against the alternative that it is lower.

     The value of the test statistic is equal to ....,

     the corresponding *p-value* is equal to ....,

     so for a significance level of 0.05 we REJECT /DO NOT HAVE GROUNDS TO REJECT (underline the appropriate) the null.
   - We verify the hypothesis that all four fractions of deposits with interest rates falling in the four ranges specified are equal.

     The value of the test statistic is ....,

     the critical value for a significance level 0.05 is equal to ....,

     so we REJECT /DO NOT HAVE GROUNDS TO REJECT (underline the appropriate) the null.