Mathematical Statistics Exam, September 2015, set B

Fill in the dotted spaces [“..........”]. 1 question (•) = 1 point; maximum = 16 points. Only responses in the specified places will be checked, without justifications or calculations. Fill in your responses **after having verified them**; if illegible or larded with corrections and crossings-out, the answers will be treated as wrong. You can use a calculator, statistical tables and one a4 sheet of paper with helpful formulas. **Communication with others is not allowed.**

NAME: .... Student’s number ....

1. Data regarding the age structure of individuals who became pensioners in 2013 (55 thousand men and 50 thousand women) was analyzed.

Age, in years	(45,50]	(50,55]	(55,60]	(60,65]	(65,70]
Share of women	0.00	0.02	0.25	0.71	0.02
Share of men	0.09	0.05	0.03	0.59	0.24

- The average retirement age calculated on the base of the above data equals to …………… for women and …………… for men, and the median retirement age equals to ………….. for women and …………… for men.
- The hypothesis that the average retirement age for men and women is equal was verified against the alternative that for women it is lower. For the studied population, the standard deviation calculated on the base of the unbiased estimator of the variance equaled to 4 years for women and 6 years for men. The value of the appropriate test statistic is equal to ………………., the critical value for the 0.01 significance level is equal to …………., so we REJECT / DO NOT HAVE GROUNDS TO REJECT the null (underline the appropriate) for this significance level.

2. Based on data from the previous problem the structure of the retirement age of women was put under scrutiny.
   - The hypothesis that the share of women who retire at age higher than 60 years is equal to 75% against the alternative that this share is lower, was verified for a significance level of 0.05. The value of the appropriate test statistic is equal to ……………., and has a *p-value* of …………., so we REJECT / DO NOT HAVE GROUNDS TO REJECT the null (underline the appropriate).
   - The first quartile of the retirement age for women is equal to ……………….. Based on this value and the results from the previous point, we can DERIVE THE SAME CONCLUSIONS / CONCLUSIONS ARE CONTRADICTORY / THE TWO POINTS ARE NOT RELATED (underline the appropriate).

3. An analyst uses the most powerful test to verify the hypothesis that the retirement age of men has a uniform distribution over the interval [55,75] against the alternative that the retirement age has a distribution with density

$$f(x) = ce^{-|65-x|}$$

for $x \in [55,75]$ and 0 otherwise, where the constant $c = e^{10}/(2e^{10}-2)$.

   - The critical region of the test is equal to …………………………....
   - The power of the test is equal to ………………………………..

4. A different analyst suspects that the retirement age of women may be described by a distribution with density

$$f_\theta(x) = \frac{6+\theta}{120} - \frac{\theta}{4000}(x-60)^2$$

for $x \in [50,70]$ and 0 otherwise, where $\theta \in (0,3)$ is an unknown parameter. The expected value of this distribution is equal to 60, and the variance is equal to $100 / 3 - 2\theta / 9$.

- Find the method of moments estimator for the parameter θ for a random sample $X_1, X_2, \ldots, X_n$ based on the second moment: ……………………………….
- Under the assumption that the *a priori* distribution of θ has density $\pi(\theta) = \frac{2}{9}(3-\theta)$ for $\theta \in (0,3)$ and 0 otherwise, find the value of the Bayesian Most Probable Estimate for θ based on a single observation $X = 59$ years: ……………………………….

5. The pension levels in cities A, B and C were analyzed based on random samples of 25 individuals from each city. The average levels equaled to 1.7, 2.0, 1.9 thousand dollars for cities A, B and C, respectively, and the standard deviation (calculated based on the unbiased estimator of the variance) amounted to one thousand dollars in each city.
   - The realization of a 95-confidence interval for the variance of the pension level in city A is equal to ……………………………..
   - Using the ANOVA test verify the hypothesis of equality of means in cities A, B and C. The appropriate test statistic is equal to ……………….., and has a distribution with …… and …… degrees of freedom. Therefore, for a significance level 0.05 we REJECT / DO NOT HAVE GROUNDS TO REJECT the null (underline the appropriate).
6. The distribution of pension levels in city A depending on sex is presented in the table below

Pension level, in dollars	(0,1000]	(1000,2000]	2000+
Number of women	48	36	36
Number of men	20	40	40

   Using the chi-squared test verify the hypothesis of the independence of pension level on sex.
   - The value of the test statistic is equal to ……………….
   - The critical region of the test for a significance level 0.05 is equal to …………………….., so we REJECT / DO NOT HAVE GROUNDS TO REJECT the null (underline the appropriate) for this level of significance.
7. The mean time of collecting pensions by men in a town was analyzed. For the studied population of 50 inhabitants, the average time of collecting pensions amounted to 15 years, with a standard deviation of 3 years (variance calculated based on the unbiased estimator). At the same time, in the general country population the average time of pension collection equaled to 17 years. We assume that the distribution of duration is normal.
   - The hypothesis that in the town under study men collect pensions on average shorter than in the general population was verified against the alternative that they collect pensions longer. The value of the appropriate test statistic is equal to ……………. For the adopted significance level of 0.05, the critical value of the test is equal to ………., so we REJECT / DO NOT HAVE GROUNDS TO REJECT the null (underline the appropriate).
   - The realization of the confidence interval for the average time spent on retirement in the studied population of men for a significance level 0,95 is equal to ………………………..
8. Let $X_1, X_2, \ldots, X_n$ be a random sample from the Pareto distribution with density

$$f_\theta(x) = \frac{\theta 50^\theta}{x^{\theta+1}}$$

   for $x > 50$ and 0 otherwise, where $\theta > 1$ is an unknown parameter.
   - Find the Maximum Likelihood Estimator of θ:……………………….
   - Let $T = \bar{X} - 50$ be an estimator of θ. Calculate the bias of T: ……………………