Fill in the gaps ["........."]. 1 question (•) = 1 point; maximum = 12 points. Write down your answer **after you have checked it**; unreadable answers will be evaluated as **wrong**! You can use a calculator, statistical tables, notes and/or textbooks. **Do not communicate; do not use mobile phones or computers; do not cheat**!

NAME: ....

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1. A survey was conducted to examine the relation between the level of education and tolerance. There were 400 people in the survey and the results were the following:

	tolerance	lack of tolerance	total
university graduate	70	30	100
high school graduate	100	100	200
no high school	30	70	100
total	200	200	400

- Compute the *test statistic* χ^2 to test for independence between the level of education and tolerance:

  $\chi^2 =$ ....

- Compute the $p - value$ and interpret the result:

  $p =$ ...., therefore *we reject/do not reject* the null hypothesis that the row and column variables are independent of each other (mark the right answer).

Hint: The $\chi^2(2)$ distribution (chi-square with 2 degrees of freedom) is the exponential distribution Ex(1/2).

2. Let $X_1, X_2, \ldots, X_n$ be *i.i.d.* random variables with probability density given by

$$f_\theta(x) = \begin{cases} \theta x^{\theta-1} & \text{for } 0 < x < 1; \\ 0 & \text{otherwise,} \end{cases}$$

where $\theta > 0$ is an unknown parameter.

- Compute the *maximum likelihood estimator* (MLE) of parameter θ, given the sample $X_1, X_2, \ldots, X_n$:

  $\hat{\theta}_{\mathrm{ML}} =$ ....

- Compute the estimator of θ by the *method of moments* (MME):

  $\hat{\theta}_{\mathrm{MM}} =$ ....

3. 2 laboratories independently measured constant c: the speed of light in vacuum. Each laboratory computed a confidence interval for c at level $1-\alpha=0.95$.

   - What is the probability that *at least one* of the two intervals contains the true value of c? ....

   - What is the probability that *both* intervals contain the true value of c? ....

4. Winnie the Pooh weighed 9 jars of honey and obtained the following results (in dag):

   $$9,\ 9,\ 8,\ 14,\ 10,\ 10,\ 7,\ 13,\ 10.$$

   Assume this is a random sample from a normal distribution $\mathrm{N}(\mu, \sigma^2)$.

   - Compute the *mean* and the *unbiased estimator of variance* from these data.

     $\bar{X} =$ ....; $S^2 =$ ....

   - Compute t-Student's confidence interval at the level of confidence

     $1-\alpha=0.95$: [..............................;..............................]

5. We observe a *single* random variable X from an exponential probability distribution $\mathrm{Ex}(\theta)$ with unknown parameter θ. We test the null hypothesis $H_0 : \theta = 2$ against the alternative $H_1 : \theta = 1$. Let δ^* be the most powerful test at the level of significance $\alpha = 0.05$.

   - $\delta^*(x) = 1$ (the test rejects null) if x satisfies the following inequality:

     ....

   - The power of δ^* is equal to ....

6. Let S be the number of successes in a Bernoulli scheme with unknown probability of success θ. It is known that $\mathbb{E}S = n\theta$ and $\mathrm{Var}S = n\theta(1-\theta)$.

   - Consider $\hat{\theta} = \dfrac{S}{n}$ as an estimator of θ and compute its variance:

     $\mathrm{Var}\hat{\theta} =$ ....;

   - Consider $\hat{\theta}(1-\hat{\theta})$ as an estimator of $\theta(1-\theta)$ and compute its bias:

     $\mathbb{E}\hat{\theta}(1-\hat{\theta}) - \theta(1-\theta) =$ ....

   Hint: To compute *the second answer* it might be helpful to use the fact that $\mathbb{E}(\hat{\theta}^2) = (\mathbb{E}\hat{\theta})^2 + \mathrm{Var}\hat{\theta}$.